Signal/noise-ratio performance of loaded wire antennas

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Abstract

Expressions for the signal/noise ratio of an antenna and of a receiving system are derived. Both signal/noise ratios are degraded by reductions in antenna efficiency and directivity. Raising an inductor from the foot of a resonant short monopole will improve the signal/noise ratio. Resistance-loaded and other travelling-wave monopoles are less directional than corresponding standing-wave structures and degrade the antenna signal/noise ratio, but the broad-band system signal/noise ratio can be improved owing to better impedance matching.

1 Introduction

Antennas may be loaded to improve their impedance match to the rest of the communication system or to modify their field patterns. If the loading is lossy, both the efficiency and the signal/noise ratio are affected. Although such an antenna may be unsuitable for transmitting, providing the sky temperature is high, its signal/noise ratio can be adequate for reception.

In this paper, several loadings, previously proposed for wire antennas,[1–9] are examined, and the performance of such antennas in receiving systems is predicted.

2 Signal/noise derivations

2.1 Antenna signal/noise ratio

The input resistance of an antenna has two components, one due to the power radiated, the radiation resistance R_r, and one due to the power dissipated in the antenna, the loss resistance R_1. If most of the loss occurs in a lumped load having a resistive component R, then R_1 is related to this value by the ratio of the squares of the antenna currents at the loading and the antenna feed.

Noise is generated at the antenna terminals and depends on the temperature of the resistances. R_r is at sky temperature, T_s and R_1 at the ambient temperature T_a. Hence, the noise power N, presented to a load having resistance R_L and reactance X_L across the antenna terminals, is

$$N = \frac{4kT_sBR_rR_L}{(R_r+R_1+R_L)^2+(X_a+X_L)^2} + \frac{4kT_aBR_1R_L}{(R_r+R_1+R_L)^2+(X_a+X_L)^2} \qquad (1)$$

where X_a is the antenna reactance.

The maximum signal power S depends on the maximum received open circuit voltage V_{oc} of the antenna and is

$$S = \frac{V_{oc}^2 R_L}{(R_r+R_1+R_L)^2+(X_a+X_L)^2} \qquad (2)$$

Hence, the signal/noise ratio $(S/N)_{ant}$ is

$$(S/N)_{ant} = \frac{V_{oc}^2}{4kB(R_rT_s+R_1T_a)} \qquad (3)$$

These quantities can be evaluated by analysis of the antenna, but further insight into the effect of loading can be found by relating these to the antenna properties of directivity and efficiency.

The directivity g_d of an antenna is its maximum directive gain compared to that of an isotropic radiator, but it can also be related to the circuit quantities used previously as

$$g_d = \frac{120\pi^2}{(E\lambda)^2}\frac{V_{oc}^2}{R_r} \qquad (4)$$

where E is the maximum incident field strength.

The efficiency η can be expressed as a resistance ratio:

$$\eta = \frac{R_r}{R_r+R_1} \qquad (5)$$

The efficiency is the same for both transmission and reception. In the transmitting case it is defined as the ratio of the power radiated to the power supplied to the antenna. For the receiving case it is the ratio of power delivered by the lossy antenna to the power delivered by a lossless antenna of the same type.

Thus, it is possible to rewrite eqn. 3 in the form:

$$(S/N)_{ant} = \frac{g_d(E\lambda)^2}{\left(1+\frac{(1-\eta)T_a}{\eta T_s}\right)480\pi^2 kBT_s} \qquad (6)$$

and comparing the antenna with a lossless isotropic antenna operating under the same conditions

$$\frac{(S/N)_{ant}}{(S/N)_{iso}} = \frac{g_d}{1+\frac{1-\eta}{\eta}\frac{T_a}{T_s}} \qquad (7)$$

Hence, if the antenna is lossless $(S/N)_{ant}$ is improved by the increase in the directivity. This is because the wanted signal is received from the optimum direction for the antenna while the noise is received from all angles. However, in many cases the sky noise is anything but uniform with direction, as noise can be received from several regions each having different temperatures. Thus, the total noise power received can be reduced further by carefully sited nulls in the antenna field pattern. In this paper, comparisons are made between the relative directivities of various antennas. This approach is valid for making signal/noise ratio comparisons, provided the significant pattern differences between the antennas are in regions over which the sky noise temperature is constant.

If lossy loading is used in the antenna, then the signal/noise ratio is degraded by the factor $1+\frac{(1-\eta)}{\eta}\frac{T_a}{T_s}$ and this factor becomes significant at low sky noise temperatures or for low antenna efficiencies.

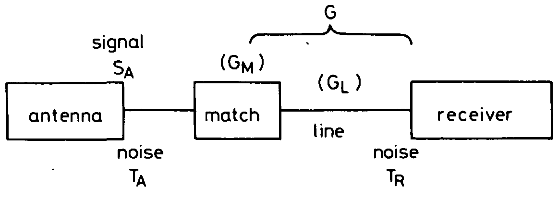

Fig. 1
Receiving system

2.2 System signal/noise ratio

The quantity $(S/N)_{ant}$ is found at a load connected directly across the antenna terminals. In a practical receiving system amplification of the signal is required and so there is a further noise contribution from the necessary following active stages. Thus, the total system signal/noise ratio $(S/N)_{syst}$ depends on the amplifier noise temperature T_R and the losses between the antenna and the amplifier.

A receiving system is shown in Fig. 1. The signal from the antenna S_A is attenuated by the mismatch G_M and line losses G_L to S_AG, while the total noise power is the sum of the attenuated antenna noise, the line noise and the receiver noise. Thus, the total noise power is given by

$$N = kT_ABG + kT_aB(1-G_L) + kT_RB \qquad (8)$$

where T_A is the equivalent antenna noise temperature,

$$T_A = \eta T_S + (1-\eta)T_a \qquad (9)$$

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and the system signal/noise ratio is

$$(S/N)_{syst} = \frac{(S/N)_{ant}}{1 + \left[\dfrac{T_R + T_a(1 - G_L)}{GT_A}\right]} \qquad (10)$$

At low frequencies sky noise is high, and unless the antenna is extremely inefficient or there is a high connection loss to the receiver or the receiver noise figure is very large, then both T_a and $T_R \ll GT_A$. Thus, for low frequencies the system noise is the same as the antenna noise.

At v.h.f. or u.h.f. the usual case is that $T_A \ll \frac{T_R}{G}$, in which case combining eqns. 6 and 10 gives the $(S/N)_{syst}$ expression as

$$(S/N)_{syst} = \frac{(E\lambda)^2}{480\pi^2 kB} \; \frac{g_d \eta G}{T_R + T_a(1 - G_L)} \qquad (11)$$

Thus, the signal/noise of a system containing a practical antenna can be found in terms of the signal/noise ratio of the same system using a lossless isotropic matched antenna as

$$\frac{(S/N)_{syst}}{(S/N)_{iso}} = g_d \eta G_M \qquad (12)$$

3 Polynomial current approximation

Popovic has shown that a point matched solution to a low order polynomial series gives a good low-order rapidly computed approximation to the current distribution on a loaded transmitting antenna.[8] This method has been used in this paper to calculate the input impedance and currents at the loading and input of the antenna. It is necessary to split the input resistance into its two components. The loss resistance is found by referring the loading resistance from the loading point to the input, by multiplying by the square of the relative currents at the two points. Its subtraction from the total input resistance yields the radiation resistance.

Appendix 9 shows how the point matched solution can be extended to the receiving antenna case and this enables the open circuit receiving voltage to be found directly from the impedance matrix used in the transmitting analysis.

4 Inductive loaded antennas

4.1 Short enhanced-radiation types

A short monopole is very capacitive, and to resonate it requires the addition of an inductor. If this inductor is located at the feed it cannot influence the current distribution, which remains approximately triangular. Raising the inductor and increasing its value to maintain resonance results in an increased current moment on the antenna, and hence a more conveniently matched input resistance. Hansen[1] studied the efficiency of this antenna deriving an expression for efficiency in terms of α, the loading/antenna reactance factor, and β, the radiation resistance improvement:

$$\eta = \frac{Q\beta R_r}{Q\beta R_r + \alpha X_a} \qquad (13)$$

For antennas of this type variations in directivity are minimal, and consequently the signal/noise ratio improvement of the raised inductor antenna over the foot loaded case is

$$\frac{(S/N)_2}{(S/N)_1} = \frac{1 + \dfrac{\left(1 - \dfrac{QR_r}{QR_r + X_a}\right)}{\left(\dfrac{QR_r}{QR_r + X_a}\right)} \dfrac{T_a}{T_s}}{1 + \dfrac{\left(1 - \dfrac{Q\beta R_r}{Q\beta R_r + \alpha X_a}\right)}{\left(\dfrac{Q\beta R_r}{Q\beta R_r + \alpha X_a}\right)} \dfrac{T_a}{T_s}} \qquad (14)$$

which can be seen to have a value of β/α when the sky temperature is low. This quantity has been evaluated[10] and shows that both peak efficiency and antenna signal/noise ratio occur when the inductor is about $0{\cdot}4\,h$ into the structure (h being the monopole height).

4.2 Short high-directivity types

Lin *et al.*[2] also considered larger inductive loads, in short monopoles: these can reverse the phase on the antenna and the result is enhanced directivity. Computations of input impedance have been made over the range of values of load reactance considered by Lin and are found to be of the same form. Differences in absolute values were found to be consistent with the usual variation between polynomial results and the King-Wu 2-term modified theory used by Lin, the polynomial solutions being closer to his experimental values.

With lossless loading, improvements in directivity of the order of 4 dB can be achieved, but the input resistance becomes very small and the gain changes rapidly with the value of the load. This increase in directivity is reduced if coils having practical Q values are used and the efficiency is also extremely poor.

It is found that these antennas are unsuitable for transmission due to their very low efficiences and this also results in the need for very high sky noise temperatures before the increase in directive gain can improve the antenna signal/noise ratio. Computations show that this loading technique is not of practical use even at the high temperatures found at low frequency.

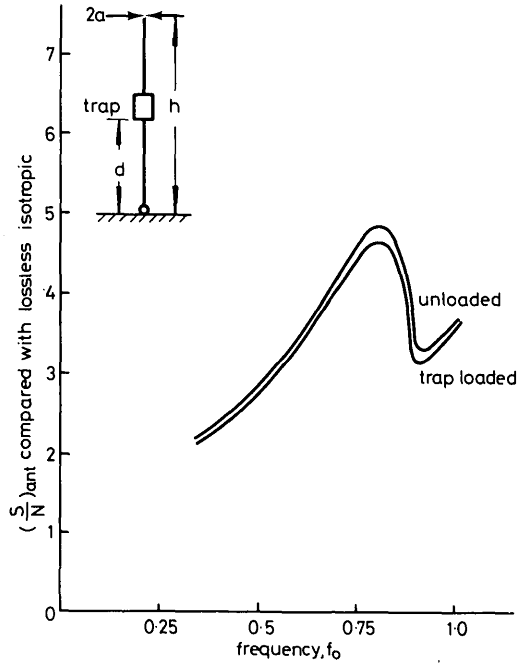

Fig. 2
Signal/noise ratio of trap loaded monopole
$h = 0{\cdot}75\,\lambda_0$, $d = 0{\cdot}25\,\lambda_0$, $a = 0{\cdot}00635\,\lambda_0$, trap impedance $= j50 \tan\left\{\frac{\pi}{2}(f/f_0)\right\}\,\Omega$

4.3 Trap loaded antennas

Parallel inductor-capacitor circuits or short circuited transmission line stubs can be designed to behave as antiresonant infinite impedance traps at a specified frequency. By using a trap at $\lambda/4$ in each arm of a $3\,\lambda/2$ dipole with traps introducing 180° phase shifts, a 3-element collinear array would be formed. Smith[3] studied such an antenna and found this phase shift was not achieved, but that the collinear pattern was set up at a lower frequency (about 0·8 times). In the equivalent monopole case the antenna is acting as $0{\cdot}6\,\lambda$ inductive loaded antenna with the load $0{\cdot}2\,\lambda$ from the end. In Fig. 2 the signal/noise ratio of this antenna is shown compared with its unloaded form. Its gain, and hence (S/N), is slightly less than the unloaded form of the same height. However, trap loaded antennas can be adjusted by varying their trap frequency or their outer section length to operate resonantly at chosen frequencies not harmonically related as is the unloaded case. Thus, for multiband working, the system signal/noise ratio on some bands may be improved by the better matching of the trap loaded antennas.

5. Travelling-wave antennas

5.1 Resistance loading

Altshuler[4] first studied a monopole antenna in which a resistor had been inserted a quarter wave from the end. A value was found at which the current below this component was of travelling-wave form.

In an approximate analysis Maclean[11] compared signal/noise ratio performance of a $3\,\lambda/2$ dipole antenna of this form with the conventional standing-wave type. His result suggested a small improvement in signal/noise ratios at high sky temperatures. From eqn. 7 this would suggest that the travelling-wave antenna was more directional. However, the zero order solution used is poor for an antenna of an integral number of half-wavelengths and more accurate computations of directivity have been made and are plotted in Fig. 3. These are for antennas with a load of 240 Ω, λ/4 from the monopole end for which Altshuler published experimental patterns and current distributions.[4] The polynomial solution is found to give good agreement with these. Thus, over this range, the gain, and in consequence the signal/noise ratio, of these antennas is less than the standing-wave antenna of the same length. In Fig. 4 the 0·625 λ antenna is taken at a reference frequency f_0 and its frequency then varied. Again, the gain is less except for a limited range around 0·885 λ. Hence, Maclean's result showing the possibility of improved antenna signal/noise ratio is confirmed, but over a narrow band only, with these slightly different antenna dimensions. However, the resistive loaded antennas have much wider bandwidths than the unloaded and noise deterioration from the following circuits can be reduced in broadband applications. The improved bandwidth can be seen in Fig. 4. The antennas are matched at f_0 and the dashed curves on the graph show the mismatches at other frequencies. Also shown are the mismatches of the resistive and unloaded monopoles into a fixed 50 Ω impedance and the efficiency of the resistive antenna. From these curves the overall system signal/noise ratio can be found across this frequency band. For the high sky-noise case, the signal/noise ratio relative to a lossless isotropic is the same as the relative directivity and so the unloaded antenna is superior over most of the range. When $T_R/G \gg T_A$, $(S/N)_{syst}$ depends on directivity, efficiency and mismatch, as indicated

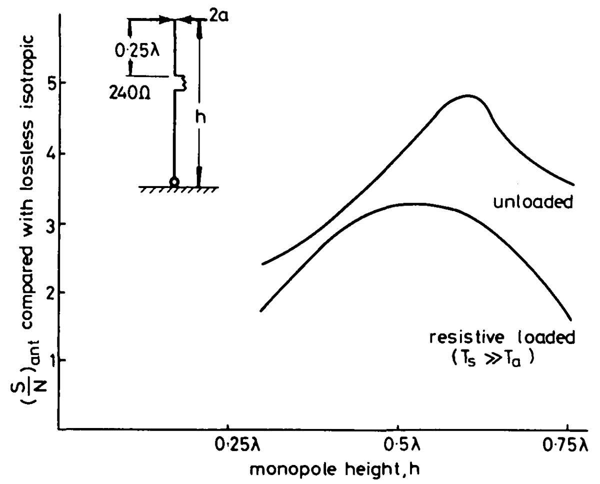

Fig. 3
Signal/noise ratio of resistive loaded monopole

$a = 0{\cdot}00635\,\lambda$

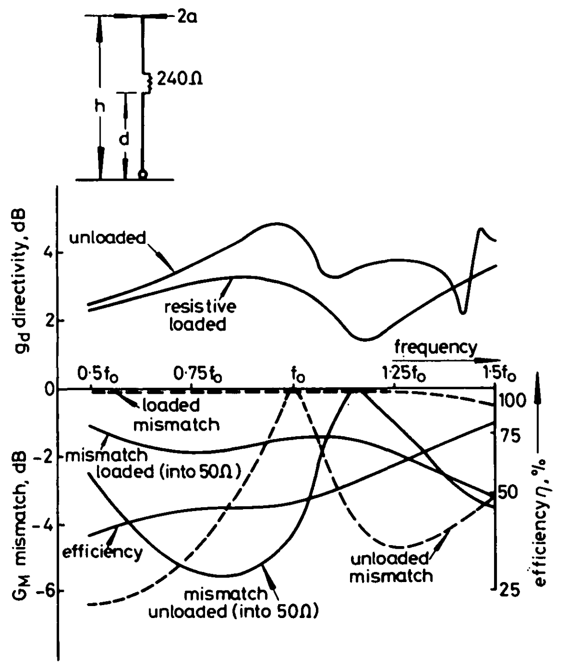

Fig. 4
Variations of directivity, mismatch and efficiency with frequency for resistive and unloaded monopoles

$a = 0{\cdot}00635\,\lambda_0,\ h = 0{\cdot}625\,\lambda_0,\ h - d = 0{\cdot}25\,\lambda_0$

in eqn. 12. The $(S/N)_{syst}$ performances of the monopoles in a 50 Ω system are shown in the upper part of Fig. 5. It can be seen that the $(S/N)_{syst}$ of the loaded antenna is more constant but inferior to the unloaded antenna even when $(S/N)_{ant}$ is greater, due to its losses and because its broadbandwidth has not been fully exploited. The lower curves, Fig. 5*b*, show the broadband system signal/noise ratio when both antennas are fed into fixed impedances, the values of which provide a match at f_0. In this case, the $(S/N)_{syst}$ is improved across a broad bandwidth by the use of the resistive monopole.

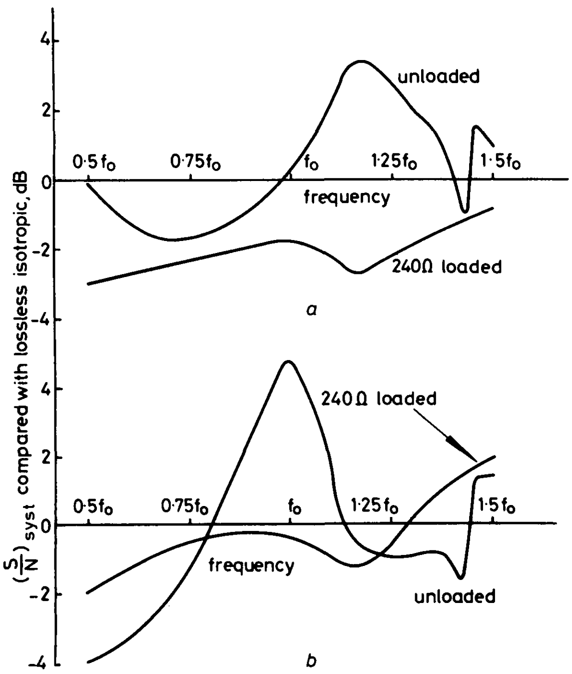

Fig. 5
System signal/noise ratio of resistive loaded and unloaded monopoles

Antenna dimensions as for Fig. 4. $T_R \gg T_A$
a System resistance = 50 Ω
b System impedance = $Z_{IN}^* @ f_0$

5.2 Ideal travelling-wave antennas

The resistively loaded antennas do not set up perfect lossless travelling waves on the lower antenna section and require the existence of a λ/4 standing-wave section, which also effects the pattern. The lumped resistor also reduces the efficiency to about 50%. However, for any length of antenna there is an optimum complex loading and optimum position for setting up a travelling wave, and at some lengths this can become wholly capacitive.[6] In Fig. 6 the directivities

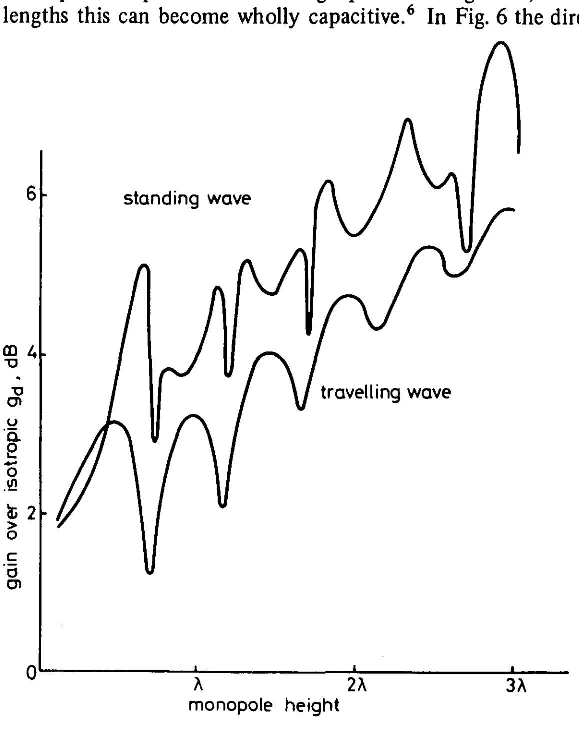

Fig. 6
Directivities of perfect standing and travelling-wave monopoles

of a perfect lossless standing wave $I_0 \sin \beta(h - z)$ and travelling wave $I_0 e^{-\beta z}$ have been calculated. These show that except for short lengths the travelling-wave antenna (over ground) has less gain than the corresponding standing-wave antenna.

At first sight, the variations in directivity for the standing-wave antenna seem surprisingly complex in form. However, the reasons for this become clear if the components of the directivity function are considered:

$$g_d = \frac{\text{maximum radiation intensity}}{\text{average radiated power}}, \tag{15}$$

The pattern produced by a single travelling wave consists of a main lobe and several backlobes. The travelling-wave monopole over ground has an image travelling wave, the backlobes of which modulate the main lobe of the first travelling wave to give a value of E_{max} which varies periodically as the monopole length is increased. The standing-wave antenna can be broken down into four travelling-wave components, and these result in a similar type of periodic variation. Thus, in both cases the maximum radiation intensity is a function which increases and varies periodically with increases in antenna height.

For the standing-wave antenna, the current moment near to the feed varies sinusoidally as the antenna length is altered. This results in a periodic variation in the radiated power (for a fixed value of maximum antenna current I_0). In the travelling-wave case the current moment and power radiated both increase smoothly with length. Dividing two increasing and periodically varying functions results in the complicated variation in standing-wave directivity shown in Fig. 6. On average, this directivity is about 1·3 dB greater than the more regularly varying directivity of the travelling-wave monopole.

Consequently, the various proposals for loading to set up a travelling-wave current result in reduced directivity and a consequent degradation in the antenna signal/noise ratio. These are the necessary penalties to achieve the desirable attributes of the travelling-wave antenna, which are its broadband character and, in the case of long antennas, a reduction in the number of minor lobes. Schemes for forming such antennas, in addition to the single lumped resistance and single lumped capacitance previously mentioned, are a distributed resistance[5] (in which the antenna is like Altshuler's about 50% efficient) and a mixture of distributed resistance and lumped capacitances[7] where the efficiency rises to about 75%. Even antennas having a series of lumped capacitors[8] result in a travelling-wave current progression with standing waves between the loads. While, if the capacitive loading is made continuous with the reactance per unit length increasing exponetially away from the feed, a decaying travelling-wave current distribution can be set up along the antenna length.[9] Hence, all of these loading schemes tend to reduce the directional gain and antenna signal/noise ratio but improve its broadband performance. Those with higher efficiency will improve the system signal/noise ratio at v.h.f. compared with Altshuler's antenna.

6 Conclusions

For an antenna loading to improve the antenna signal/noise ratio it must introduce directional gain. The overall signal/noise ratio of a receiving system also depends on noise contributions from its receiver and interconnecting line. Thus, it is possible to improve overall system signal/noise ratio by better impedance matching even when the antenna signal/noise ratio is degraded.

The optimum loading point for a resonant coil loaded short monopole is about 0·4 h. Schemes for increasing the directivity of short antennas by large inductive loads lead to very poor efficiency and deterioration in the signal/noise ratio. Longer antennas loaded by inductor-capacitor traps have reduced directivity compared with unloaded dipoles but enable specified multiband working to be achieved.

Comparisons between standing-wave and travelling-wave antennas show a degradation in directivity, and hence also antenna signal/noise ratios for the travelling case. Travelling-wave antennas have broader bandwidths and approximations to them are set up by resistive and capacitive loading schemes. Their use in broadband systems can result in signal/noise ratio improvement owing to the better matching.

7 Acknowledgments

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9 Appendix

9.1 Polynomial currents

In Popovic's method, the vector potential A_z, at any point z on the antenna axis, is found so that it satisfies the condition $E_{tangential}$ is zero outside the generator region. For an unloaded antenna this gives

$$A_z(z) = C \cos \beta z + \frac{\beta V}{j2\omega} \sin \beta |z| \tag{16}$$

where V is the magnitude of a delta function generator at $z = 0$ and C is a constant to be determined.

This is then equated to the vector potential of the unknown current $I(z)$

$$A_z(z) = \frac{\mu_0}{4\pi} \int_{-h}^{h} I(z') \frac{e^{-j\beta r}}{r} \, dz' \tag{17}$$

for a dipole antenna of length $2h$.

However, if the antenna is also irradiated by an incident field E_z, of constant value along the z-axis, then the total vector potential expression is modified to

$$A_z(z) = \frac{\mu_0}{4\pi} \int_{-h}^{h} I(z') \frac{e^{-j\beta r}}{r} \, dz' + \frac{E_z}{\omega} \tag{18}$$

$I(z')$ can be replaced by a polynomial series of order n as an approximation to the current, and by including the boundary condition of zero current at the antenna end a suitable form is

$$I(z') = \sum_{m=1}^{n} I_m \left(1 - \frac{|z'|}{h}\right)^m \tag{19}$$

where I_m are complex constants to be determined.

Substituting this series and equating the two expressions for $A_z(z)$ gives

$$\sum_{m=1}^{n} I_m \int_{-h}^{h} \left(1 - \frac{|z'|}{h}\right)^m \frac{e^{-j\beta r}}{r} \, dz' - C \cos \beta z$$

$$= \frac{-E_z}{\omega} + \frac{V}{j60} \sin \beta |z| \tag{20}$$

To determine coefficients of the series $n + 1$ complex equations of the form

$$\sum I_m F_m(z_p) - C \cos \beta z_p = -\frac{E_z}{\omega} \text{ receiving} \tag{21}$$

and

$$= \frac{V}{j60} \sin \beta |z_p| \text{ transmitting} \tag{22}$$

are written and the matrix is inverted to give the current coefficients as either functions of applied voltage or incident field.

In the above cases the current is symmetrical, and matching points are only required over half the antenna. If it is required to find the short circuit receiving current for different directions of incident field the distribution is no longer symmetric and a matrix of twice the size is required. Consequently, it is faster to use the one matrix for both transmitting and receiving cases and to find the far-field pattern from the transmitting-current solution

$$E_\theta = \frac{\eta \sin\theta e^{-j\beta r}}{4\pi r}\int_{-h}^{h}\sum_{m=1}^{n} I_m\left(1-\frac{|z'|}{h}\right)^m e^{j\beta z'\cos\theta}dz' \qquad (23)$$

as these integrals can be expressed in closed form.

Thus, the received open circuit voltage is found by multiplying the receiving short circuit current by the input impedance found from the transmitting case. Its maximum value is then found by using the transmitting-radiation pattern.

The loaded antenna requires separate current series for each section and additional voltage generators for each load Z_L at height z_d modify the vector potential expression of eqn. 16 by additional terms of the form of

$$A_z(z) = \frac{-\beta}{j2\omega} Z_L I(z_d)(\sin\beta|z-z_d| + \sin\beta|z+z_d|) \qquad (24)$$

This is shown in Popovic's paper on lumped impedance loadings in cylindrical antennas[8] and merely results in modifications to the elements $F_m(z_p)$ of the matrix eqns. 21 and 22.

For the computed results in this paper the antennas were subdivided into sections of less than a quarterwave and second-order polynomial solutions fitted by using equispaced matching points along these sections.