Radiated Power From Verticals

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Feedpoint Equivalent circuit model

Figure 1 shows the traditional equivalent circuit used to represent the resistive part of an antenna's feedpoint impedance (Ri) when describing what happens to the input power Pi. Rr is the radiation resistance which represents the radiated power $Pr=Io^2Rr$ where Io is the current at the feedpoint in Arms. Rg accounts for the power lost in the soil close to the antenna. RL represents the sum of other ohmic losses such as conductor loss, insulator leakage, etc. The input resistance at the feedpoint is assumed to be the sum of these resistances, i.e. Ri **=** Rr + Rg + RL.

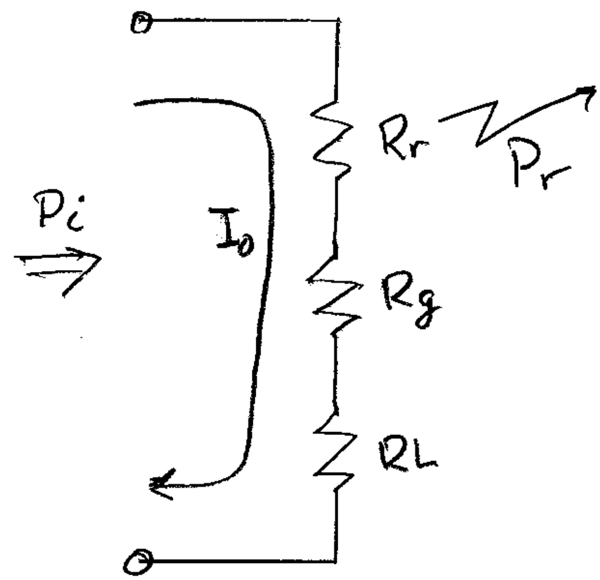

Figure 1 - Typical equivalent circuit for the feedpoint resistance.

Determining PL is reasonably straightforward but Pg is trickier. In the following discussion I will be ignoring RL, i.e. lossless conductors will be assumed. This is not because these losses are unimportant but the interest here is in Rr and Rg and how they vary with frequency, ground system design and soil characteristics. PL is certainly worthy subject for another day.

The traditional assumption has been that Rr for a vertical over real ground is the same as for the same antenna over perfect ground. The value we measure for Ri is assumed to be the sum of the Rr for perfect ground and additional loss terms due to ground and other loss elements. I've certainly gone along with the conventional thinking but over the years I've become skeptical after seeing experimental and modeling results and calculations that didn't fit. I've come to the conclusion that at HF at least, Rr for a given vertical over real soil, is not the same value for the same antenna over perfect ground. The following discussion focuses on the concept illustrated in figure 1 with RL=0. The discussion will show that both Rr and Rg vary with frequency, ground system design and/or soil characteristics.

To make this article easier to read I've placed almost all the mathematics and the many supporting technical details in an extensive set of appendices.

Appendix A - Rr calculation using the Poynting vector

Appendix B - A review of soil characteristics

Appendix C - E & H fields and power integration

Appendix D - Miscellaneous bits

Pushing material into appendices makes life much easier for the casual reader but provides the gory details for those who want them. These appendices are available on my web site: www.antennasbyn6lf.com and on the QEX web site TBD.

Rr for a lossless antenna

We need to be careful with our use of the term "radiation resistance". A definition of Rr associated with a lossless antenna in free space, can be found in almost any antenna book. A typical example is given in Terman[1]:

"*The radiation resistance referred to a certain point in an antenna system is the resistance which, inserted at that point with the assumed current I_o flowing, would dissipate the same energy as is actually radiated from the antenna system. Thus*

$$\boldsymbol{Radiation\ resistance = \frac{radiated\ power}{I_o^2}}$$

Although this radiation resistance is a purely fictitious quantity, the antenna acts as though such a resistance were present, because the loss of energy by radiation is equivalent to a like amount of energy dissipated in a resistance. It is necessary in defining radiation resistance to refer it to some particular point in the antenna system, since the resistance must be such that the square of the current times radiation resistance will equal the radiated power, and the current will be different at different points in the antenna. This point of reference is ordinarily taken as a current loop, *although in the case of a vertical antenna with the lower end grounded, the grounded end is often used as a reference point*."

Discussions of Rr for the lossless case are common but I've not seen discussion of Rr where the effect of near-field losses are considered. Kraus[2] does tease us with a comment:

"*The radiation resistance Rr is not associated with any resistance in the antenna proper but is a resistance coupled from the antenna* <u>and its environment</u> *to the antenna terminals.*"

The underline is mine! The implication that the environment around the antenna plays a role is important but unfortunately Kraus does not seem to have expanded on this observation.

<u>Calculation of Rr and Rg</u>

As pointed out earlier if you know Io and Pr you can calculate Rr. A standard way to calculate the total radiated power is to sum (integrate) the power density (in W/m^2) over a hypothetical closed surface surrounding the antenna. For lossless free space calculations the enclosing surface can be anywhere from right at the surface of the antenna to an sphere with a very large radius (large in terms of wavelengths). For Pr calculations a large radius has the advantage of reducing the field equations to their far-field form which greatly simplifies the math. This is fine for lossless free space or over perfect ground, where near-field or far-field values give the same answer. However, when we add a lossy ground surface in close proximity to the antenna things get more complicated. Note, the terms near-field, Fresnel and far-field are carefully defined in appendix C.

Take for example a vertical λ/2 dipole with the bottom a short distance above lossy soil. You could create a closed surface which surrounds the antenna but does not intersect ground and then calculate the net power flow through that surface. When you do this you find the Ri provided by EZNEC[3] (my primary modeling software) will be the same as the Rr calculated from the power passing through the surface. Technically this is Rr by the free space definition since the antenna is lossless as is the space within the enclosing surface, but that's not how we usually think of the relationship between Ri and Rr. The conventional point of view is that the near-field of the antenna induces losses in the soil which we assign to Rg, separate from Rr as indicated in figure 1. The power absorbed in the soil near the antenna is not considered to be "radiated" power although clearly it is being supplied from the antenna. When we run a model on NEC or make a direct measurement of the feedpoint impedance of an actual antenna, we get a value for Ri= Rr + Rg.

Can we separate Rr from Rg and if so, how? Assuming we're going to use NEC modeling, we could simply use the average gain calculation (Ga). The problem with Ga is that it includes all the ground losses, near and far-field, ground wave, reflections, etc. For verticals Ga gives a realistic if depressing estimate of the power radiated for sky wave communications but the far-field loss is not usually included in Rg. Typically Rg represents only the losses due to the reactive near-field interaction with the soil. In the case of a λ/4 ground based vertical for example, that would be the ground losses out to ≈λ/2 (see appendix C). Instead of using Ga we can have NEC give us the amplitudes and phases of the E and H fields on the surface of a cylinder which intersects the ground surface as indicated in figure 2.

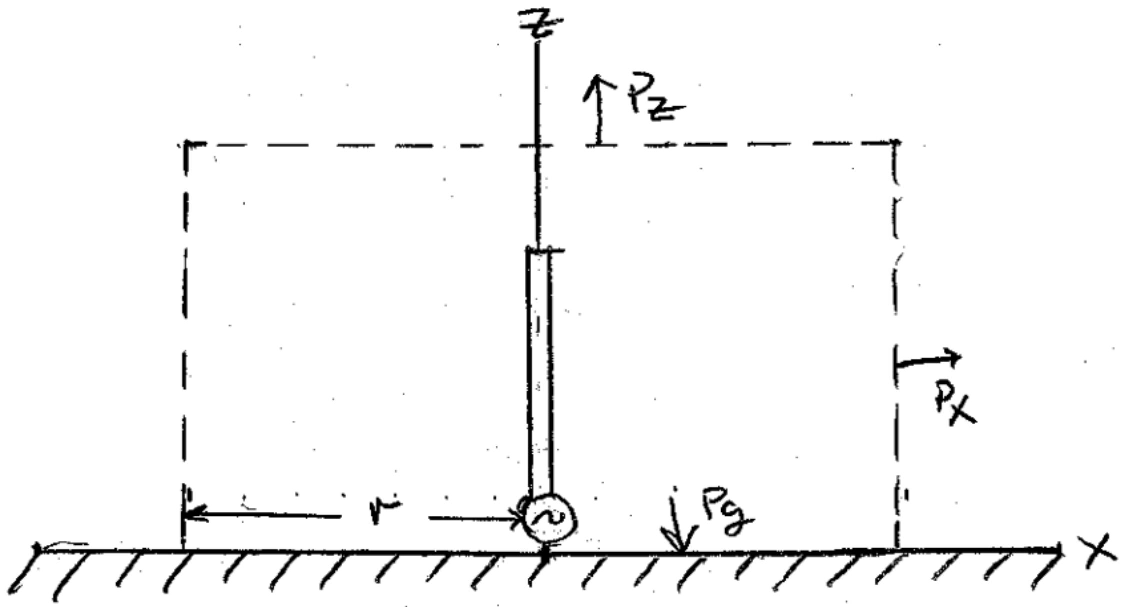

Figure 2 - cylindrical surface enclosing a ground mounted vertical.

The power density is integrated over the surface of the cylinder (Px) and over the surface of the disc (Pz) which forms the top of the cylinder giving us Pr directly. Instead of integrating the power over the surface of the cylinder we could sum the power passing through the soil interface at the bottom of the cylinder which gives Pg directly. From either Pr or Pg we can calculate Rr: $Rr = Pr/Io^2 = (Pi-Pg)/Io^2$. Of course this is more complicated that simply using Ga! But it turns out if you're moderately clever in your choice of surface and field components to be quite practical using a spreadsheet like EXCEL. The mathematical details are in appendix A. Because the fields near a vertical are sums of decaying exponentials ($1/r$, $1/r^2$, $1/r^3$) the boundaries between the field regions are not sharply defined, the choice for the cylinder or disc radius is somewhat arbitrary. The rather messy details of the choice of integration surface radius are discussed in appendix C.

Rr and Rg for a λ/2 vertical dipole

For simplicity I began the study using a resonant vertical λ/2 dipole like that shown in figure 3 with the bottom of the antenna was 1m above ground. The analysis was done at several frequencies two of which are reported here, 475 kHz and 7.2 MHz. Note the frequencies are a factor of ≈16X apart. In a later section I give an example at 1.8 MHz. The antennas heights (h) were adjusted for resonance over perfect ground and that height was retained for modeling over real soil.

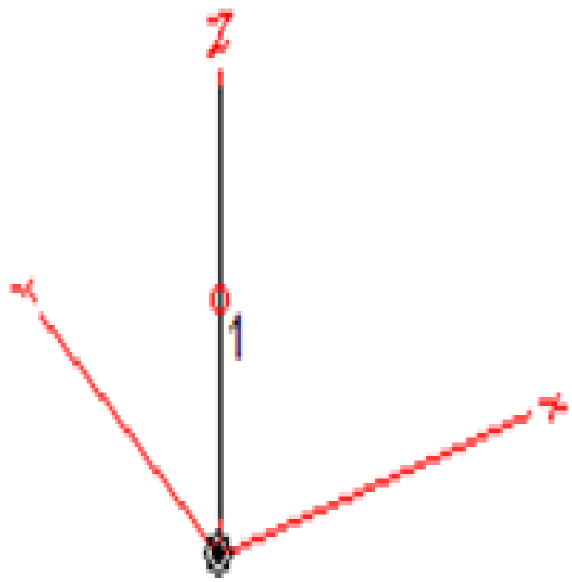

Figure 3 - Vertical dipole.

Figures 4 and 5 show the variation in Ri at 7.2 MHz and 475 kHz for a wide range of soil conductivity (σ) and permittivity (ϵr, relative dielectric constant). The notation "J=" on the figures indicates the height of the bottom of the antenna above ground.

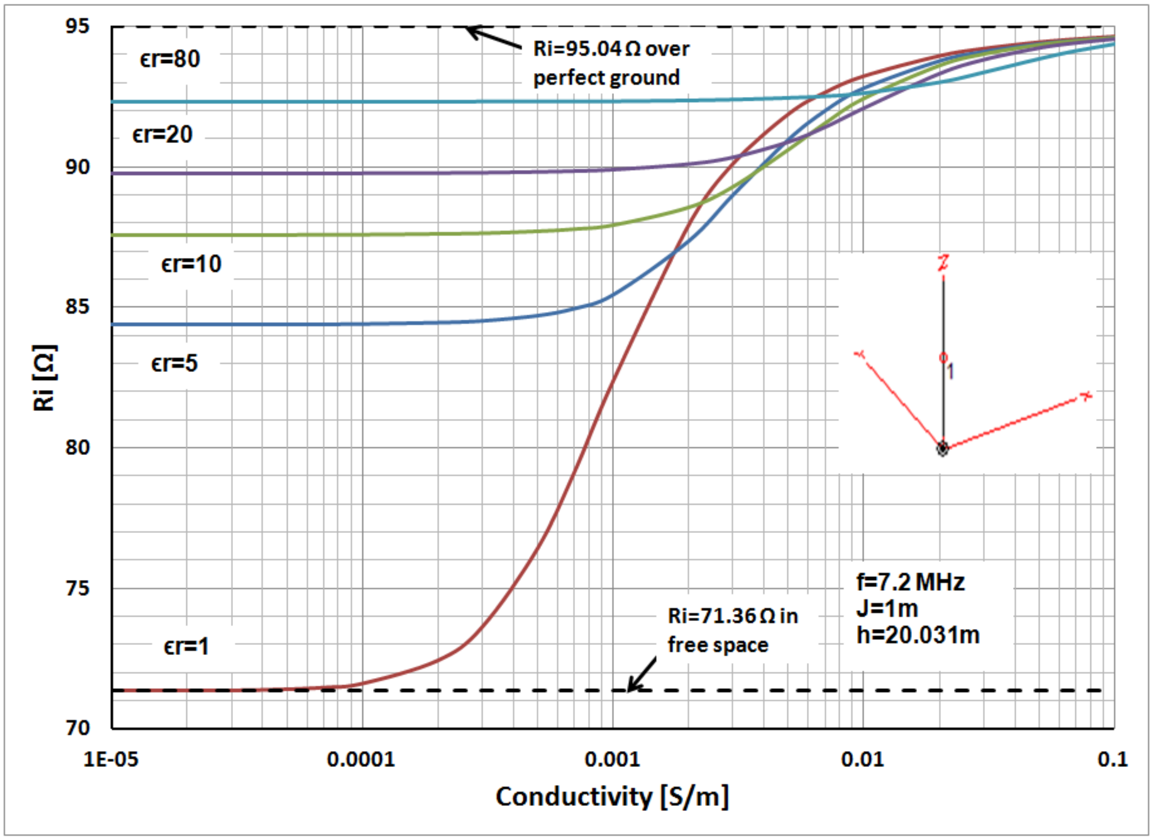

Figure 4 - Ri for a vertical dipole at 7.2 MHz.

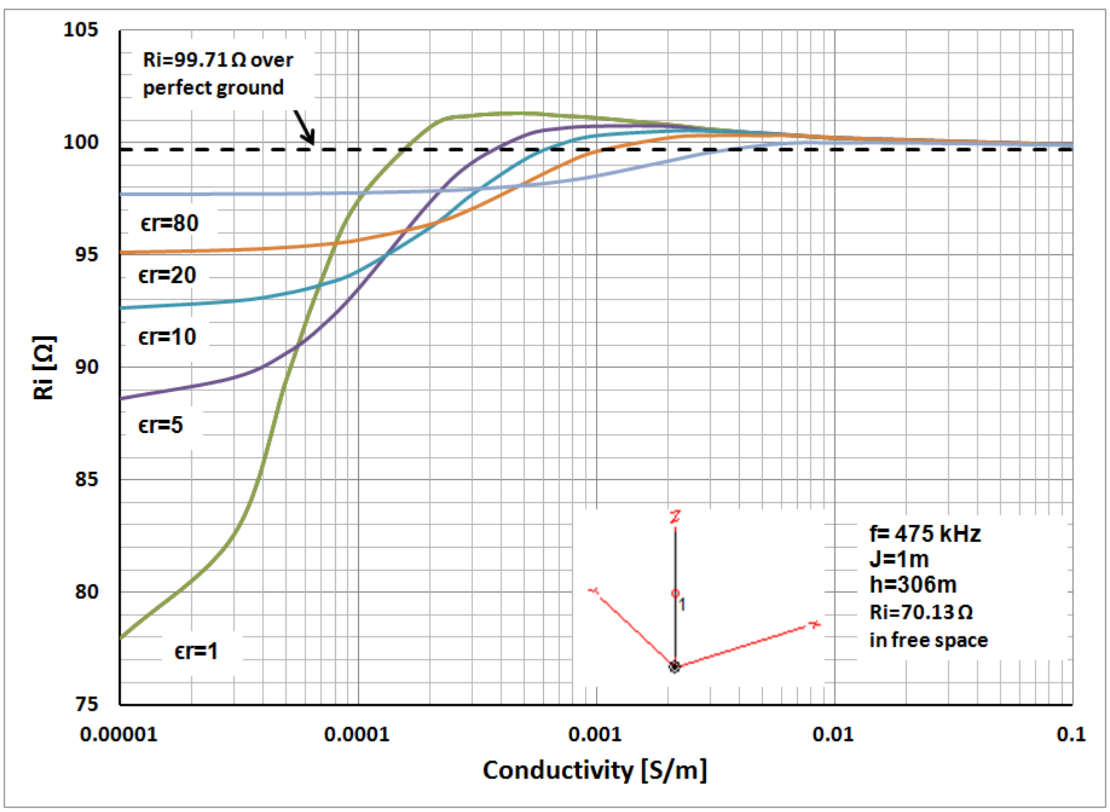

Figure 5 - Ri for a vertical dipole at 475 kHz

As we would expect, in free space Rr≈72Ω and over perfect ground Rr≈95-100Ω for these antennas. Over real ground Ri varies dramatically with both soil characteristics and frequency. One point is obvious:

Ri is not a combination of Rr over perfect ground and some Rg!

Note that for some values of σ, Ri is greater than it's perfect ground value. By the traditional view this implies Rg is negative which is not physically reasonable! On 40m values for Ri over real soils are all lower than the perfect ground case but the values on 630m vary from well below the perfect ground case to slightly above. In both cases as ground conductivity increases Ri converges on the perfect ground case as one would expect. For very low conductivities we can see that ϵr has a profound influence on Ri but it's effect is greatly reduced for high conductivities. Note that at 475 kHz for σ≧0.0001 S/m Ri rapidly converges on the perfect ground value and the effect of ϵr is minimal. On the other hand at 40m the jump in Ri doesn't occur until σ≧0.003 S/m, that's more than an order of magnitude higher than 475 kHz. It would appear that at 475 kHz the value for ϵr doesn't matter much over most common soils but at 7.2 MHz it has a major influence for some typical values of σ. What's going here?

Soil characteristics

It's important to understand that the characteristics of a given soil will vary with frequency. The following is a brief overview, a much more detailed discussion can be found in appendix B. Figures 6 and 7 are examples of σ and ϵr for a typical soil over a frequency range from 100 Hz to 100 MHz. These graphs were generated using data excerpted from King and Smith[4]. In this example at 100 Hz σ≈0.09 S/m and that value is relatively constant up to 1 MHz beyond which σ increases rapidly. ϵr behaves just the opposite, decreasing with frequency until about 10 MHz and then leveling out. We can combine σ and ϵr by using the loss tangent (D).

$$D = \tan\delta = \frac{\sigma_e}{2\pi f \varepsilon_e}$$

$\varepsilon_e = \varepsilon_o \varepsilon_{er}$ = effective permittivity or dielectric constant [Farads/m] and ε_o = permittivity of a vacuum = 8.854 X 10^{-12} [Farads/m]. For a good insulator D<<1 and for a good conductor D>>1. For most soils at HF 0.1<D<10 but it is often close to 1.

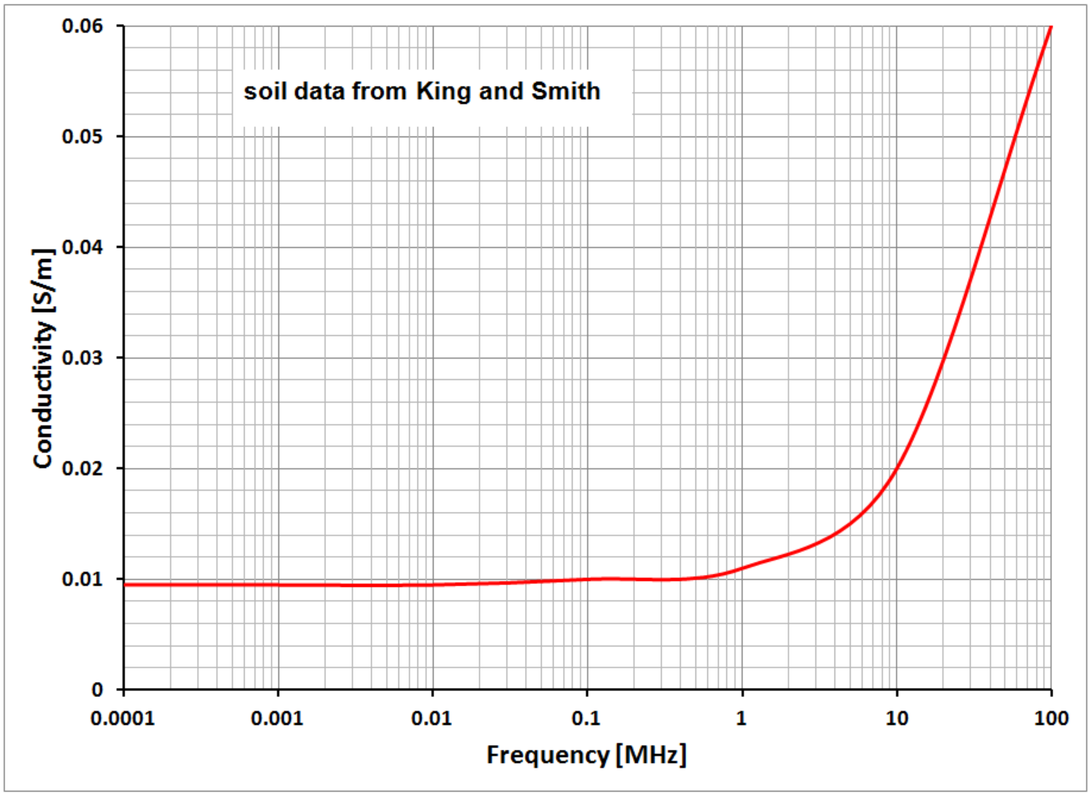

Figure 6 - Example of soil conductivity variation with frequency.

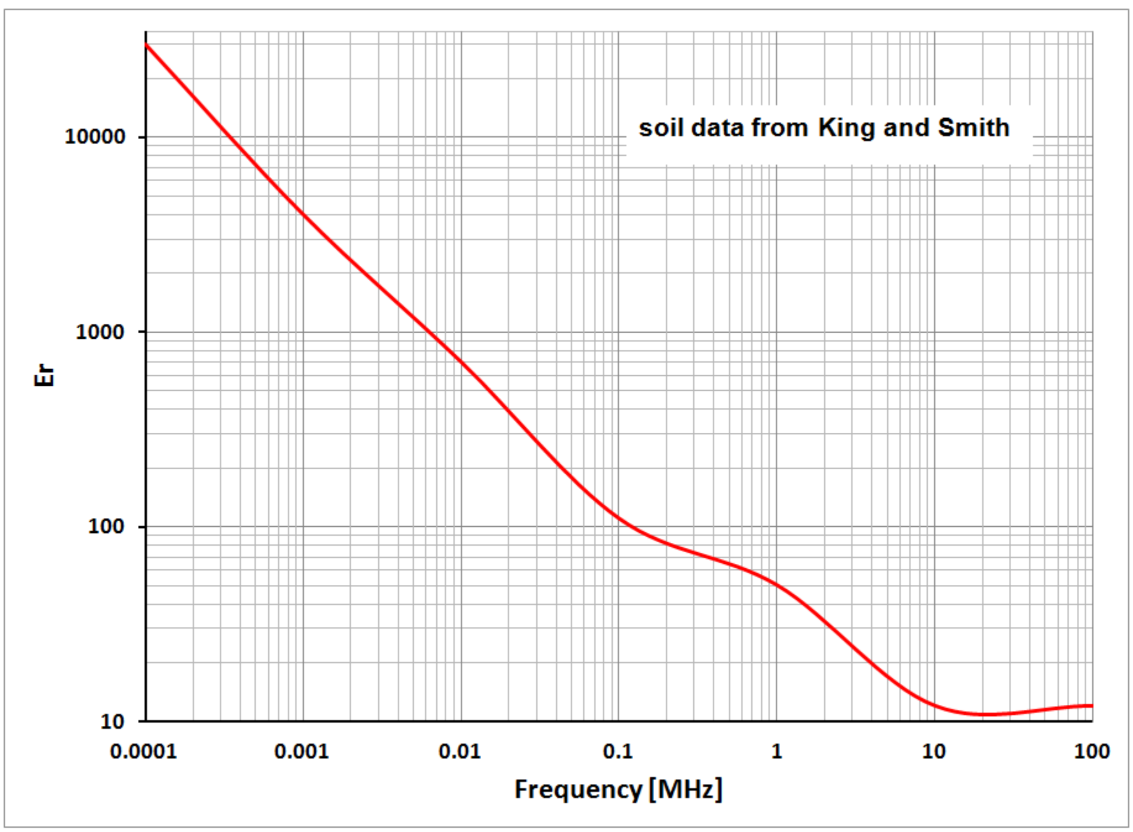

Figure 7 - Example of soil permittivity variation with frequency.

We can combine the data in figures 6 and 7 into a graph for D as shown in figure 8.

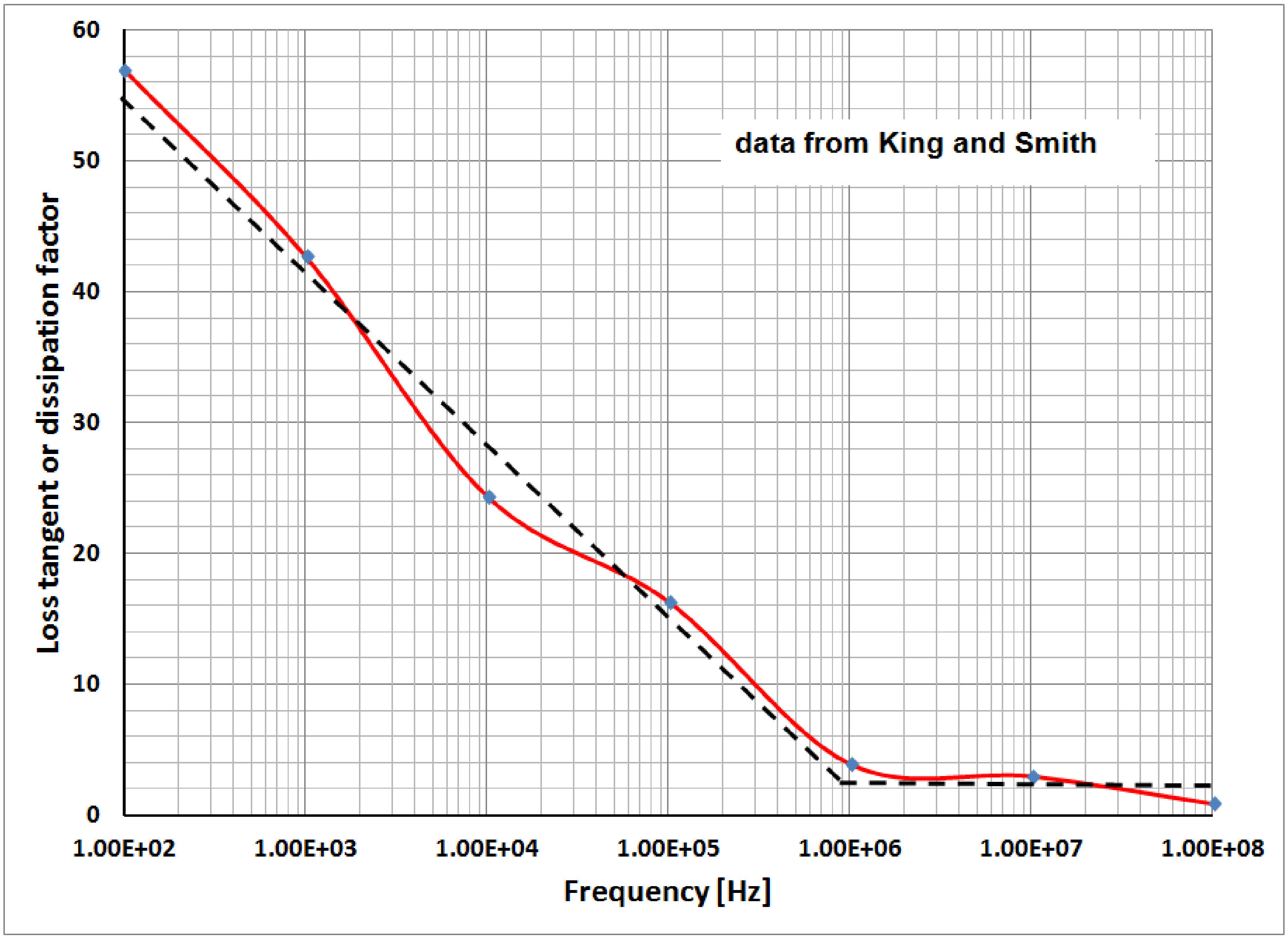

Figure 8 - Graph of the loss tangent associated with the soil in figures 6 and 7.

Figure 8 shows that something interesting happens when we go from HF down to MF. At HF D is usually not far from 1 but at MF D is usually much higher which implies the soil characteristics are dominated by conductivity. Figures 4 and 5 show that at MF conductivity becomes the dominant influence at much lower conductivities than at HF. This explains some of the features of figures 4 and 5.

Relationships between D, Rr and Rg

The role of the loss tangent D is worth exploring a bit further. Figure 4 showed the variation in Ri as ϵr and conductivity were varied. In a similar way we can examine the variation in Rr and Rg over the same range of variables as shown in Figure 9 which is a graph of Ri, Rr and Rg with ϵr =10 for the 40m λ/2 vertical. On the chart there is vertical dashed line corresponding to values of σ where D=1 for ϵr=10 (σ≈0.004 S/m in this example). Something interesting happens in the region around the point where the loss tangent equals one.

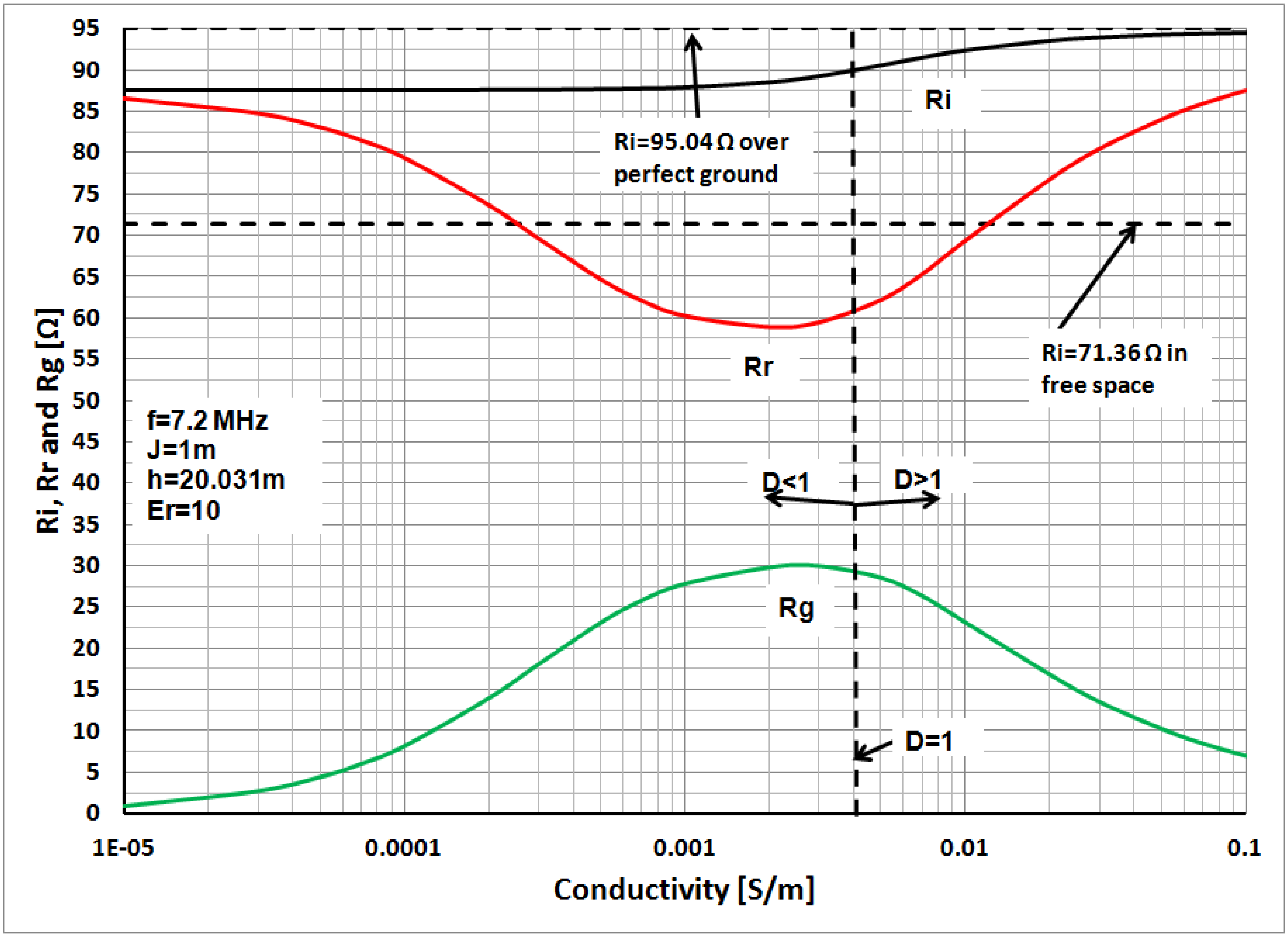

Figure 9 - Variations in Ri, Rr and Rg with Er=10.

A very prominent feature of figure 9 is that Rr and Rg are <u>not</u> constant as we vary σ. The value for Rg (which represents ground loss) peaks near D=1 which is what dielectric theory predicts for the maximum dissipation point. We can take one further step with the data in figure 9 and graph the ratio Rr/Ri (which is the radiation efficiency) as shown in figure 10. The minimum efficiency (≈0.66) occurs at σ≈0.0025 S/m.

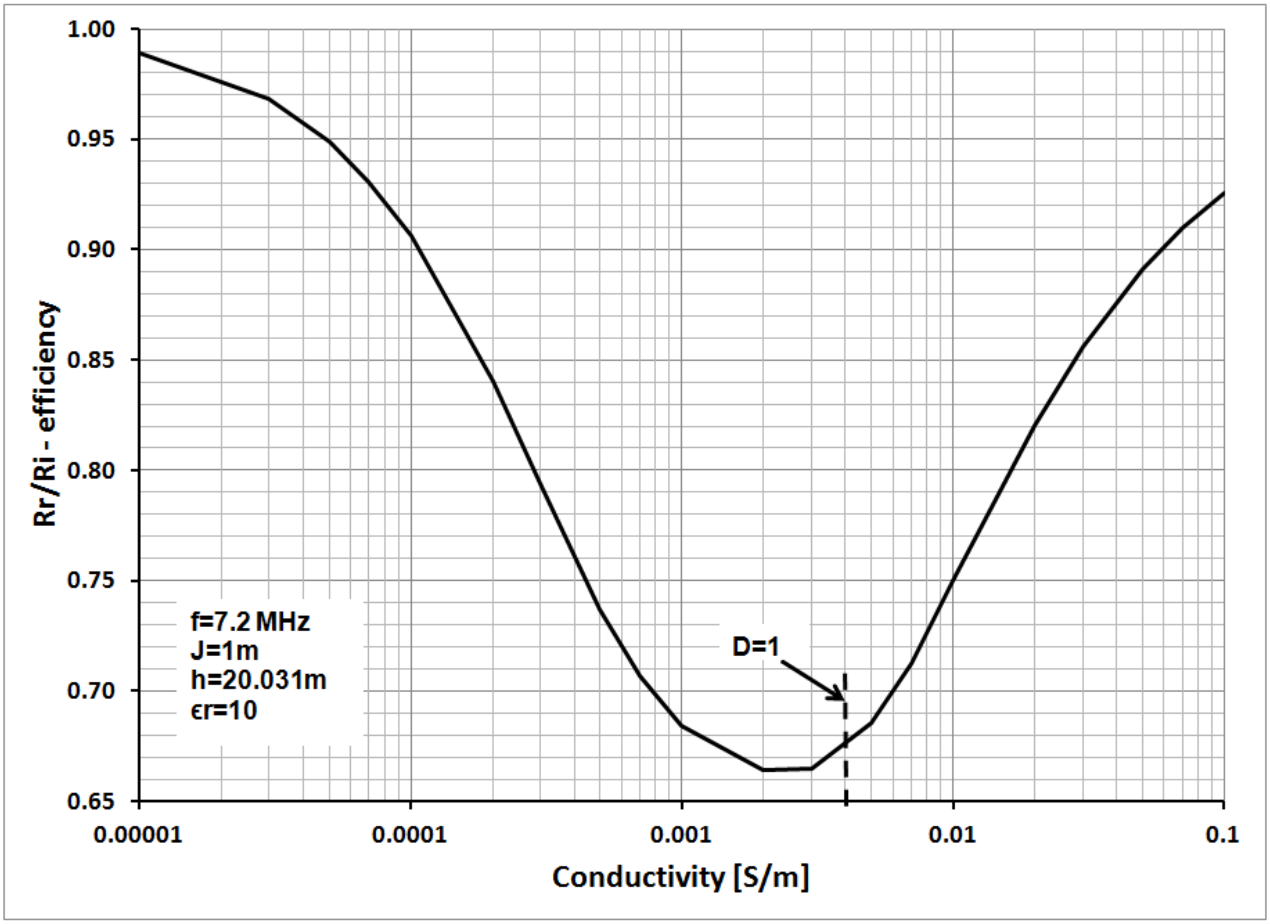

Figure 10 - Radiation efficiency with ϵr=10.

This graph emphasizes the effect of the loss tangent on ground loss.

Soil-antenna interaction

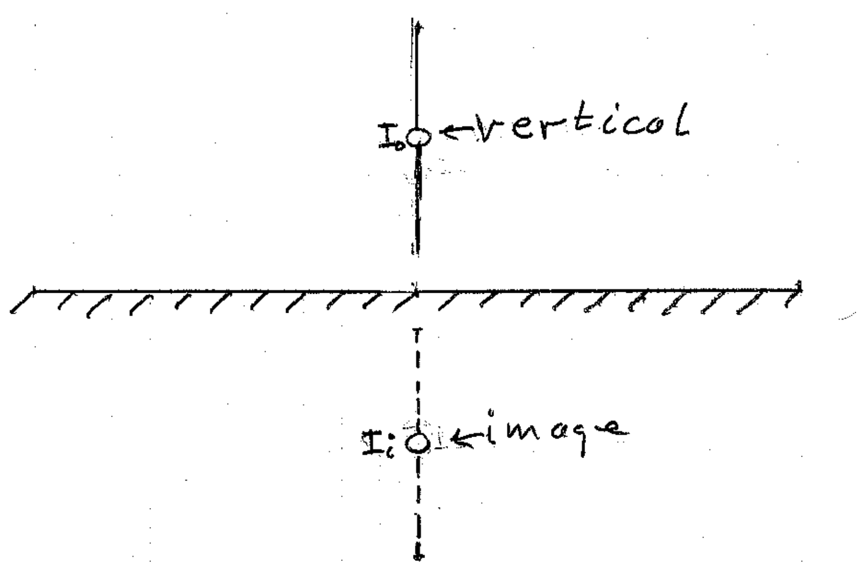

Figure 11 - example of an antenna image.

As illustrated in figure 11, one way to analyze a vertical over ground is to use a hypothetical image. If the ground is perfect then the image antenna will be a duplicate of the actual antenna with the same current amplitude and phase. For a dipole a short distance above ground the image is another dipole the same distance below ground. We now have a system of two coupled dipoles and it's no surprise that Ri of the upper dipole is no longer ≈72Ω but in these examples Ri≈94-100Ω. What's happening is that the upper vertical (the real one) has a self resistance of ≈72Ω but added to that is a mutual resistance (Rm) coupled from the image antenna.

However, if the ground is not perfect then the image antenna will not be a exact replica of the real antenna. The current amplitude and phase on the image will be different so we should not be surprised if Ri does not have the same value as either the free space or perfect ground cases. Viewing Ri as a combination of the free space value and some mutual ±Rm due to the soil is perfectly valid and this was Wait's approach[6] 5where he calculated the **±Δ**Ri as the soil and/or radial fan is changed. But his **±Δ**Ri was a combination of changes in Rr and Rg not Rg alone.

Rr and Rg for a λ/4 vertical at 7.2 MHz

The λ/4 vertical with a buried radial screen shown in figure 12 is more representative of typical amateur antennas for 40m than a full height λ/2 vertical dipole. However, amateurs are not likely to use a full λ/4 vertical on 630m which would be ≈500' high! We'll look at a more typical 630m antenna in a later section.

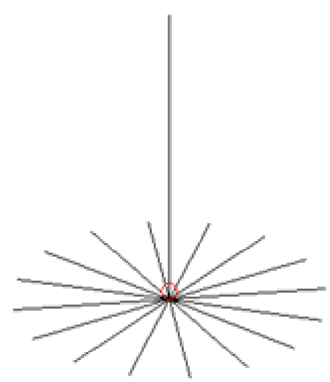

Figure 12 - λ/4 vertical with a buried radial screen.

I calculated data points for 16, 32 and 64 radials, with lengths of 2, 5, 10 and 16m over poor (0.001/5), average (0.005/13) and very good (0.03/20) soils and figure 13 is a graph showing the behavior of Ri, Rr and Rg as a function of radial length when 64 radials are employed over average ground at 7.2 MHz.

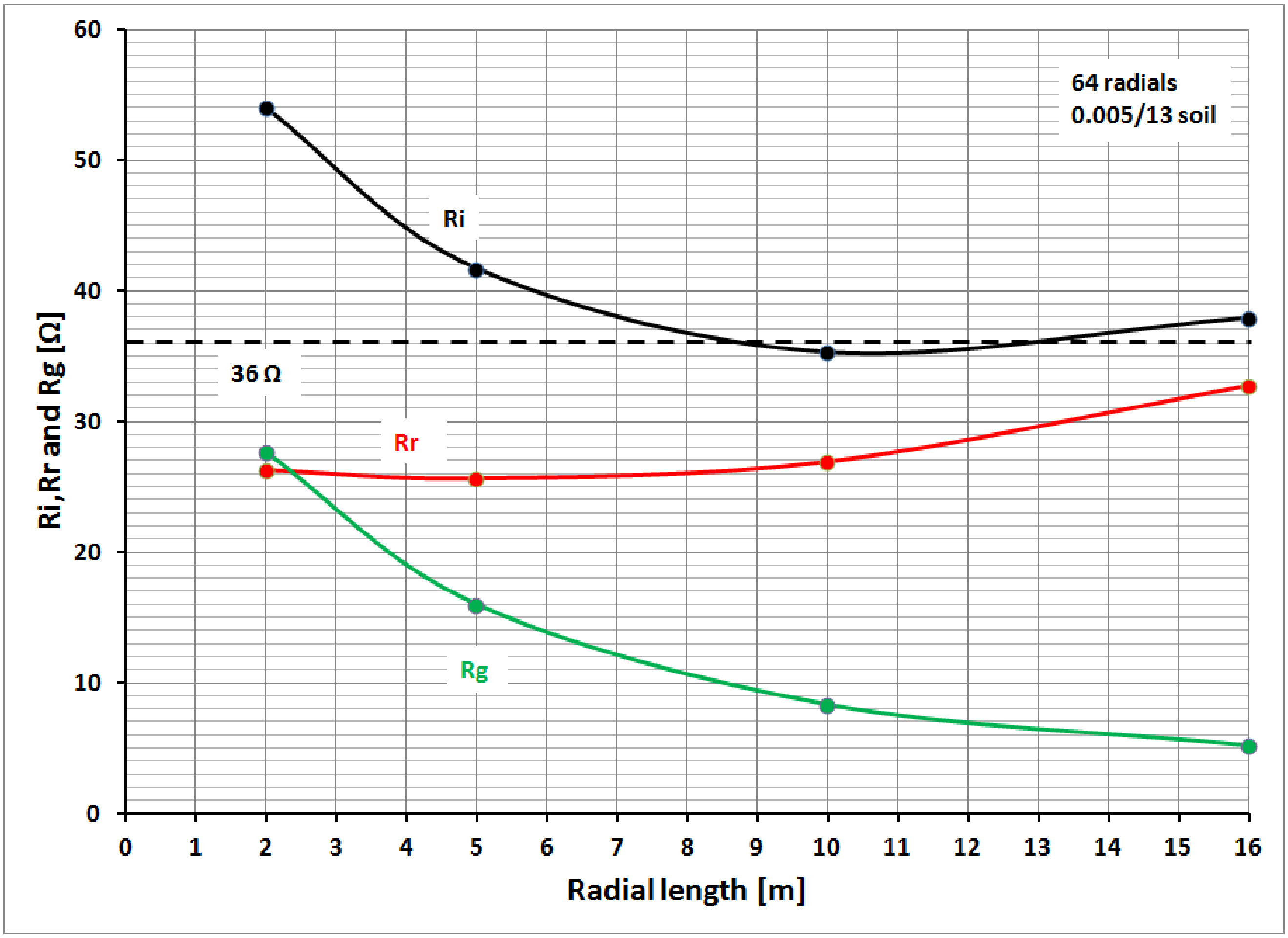

Figure 13 - Ri, Rr and Rg as a function of radial length for a 40m λ/4 vertical.

On the graph there is a dashed line labeled "36Ω" corresponding to the value of Rr for a resonant λ/4 vertical over infinite perfect ground.

The fact that Ri does not decrease or even flatten out for radial lengths >λ/4 but instead starts to increase has been predicted analytically (for example Wait[6]), my earlier NEC modeling (see appendix D) and seen in practice. What's interesting is that Rr≠36Ω! Rr starts out well below the value for an infinite perfect ground-plane but as the radial length is extended it approaches 36Ω. Increasing the radial number and/or extending radial length also moves Rr closer to 36Ω. Figure 13 represents only one case: 64 radials over average ground.

Figure 14 gives a broader view of the behavior of Rr for different soils and radial numbers as radial length is varied.

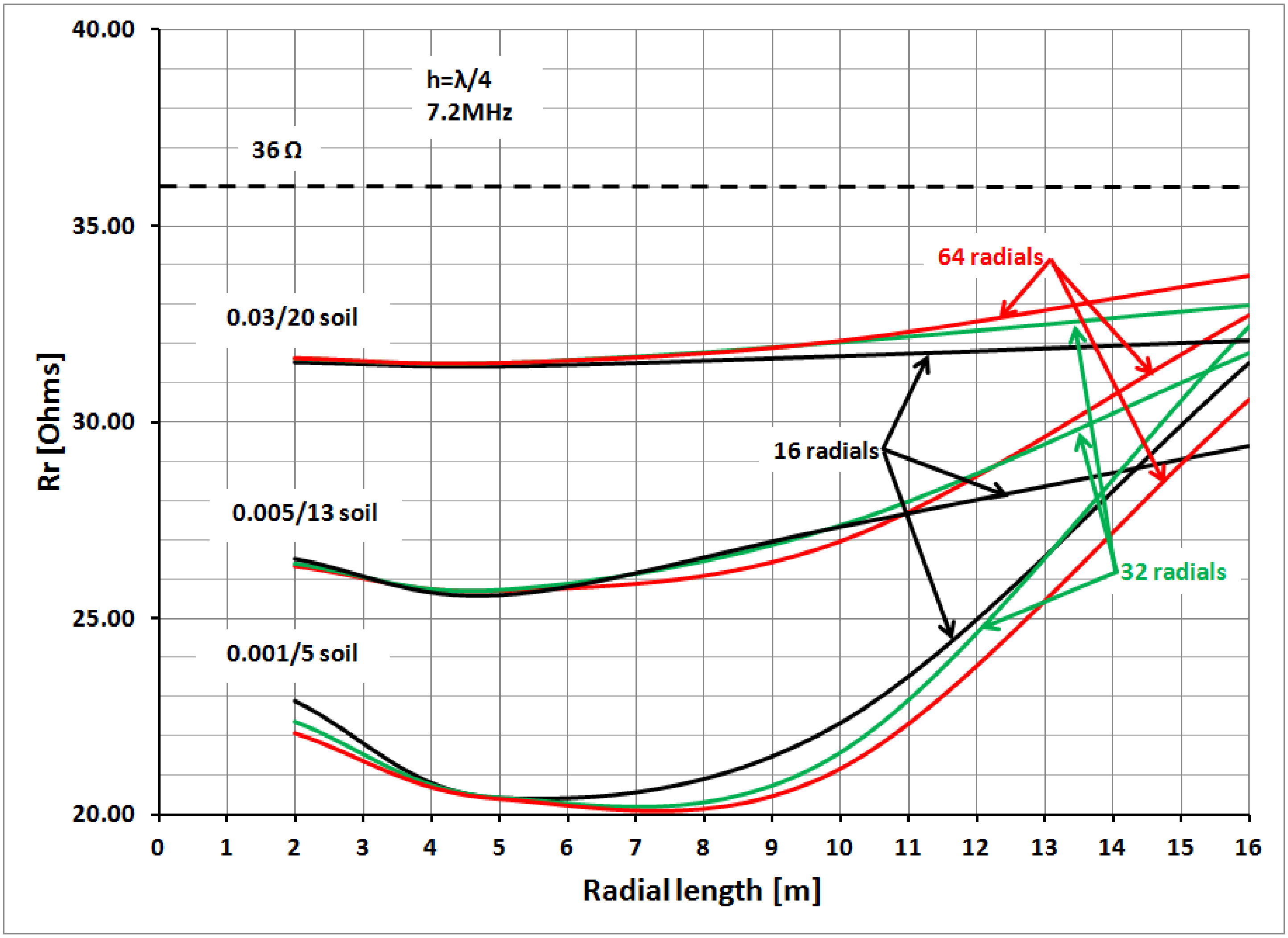

Figure 14 - Rr as a function of soil, radial number and radial length.

It's abundantly clear that Rr≠36Ω but as we improve the soil conductivity and/or increase the number and/or length of the radials Rr converges on 36Ω. We can also graph the values for Rg as shown in figure 15 which nicely illustrates how more numerous and longer radials reduce ground losses.

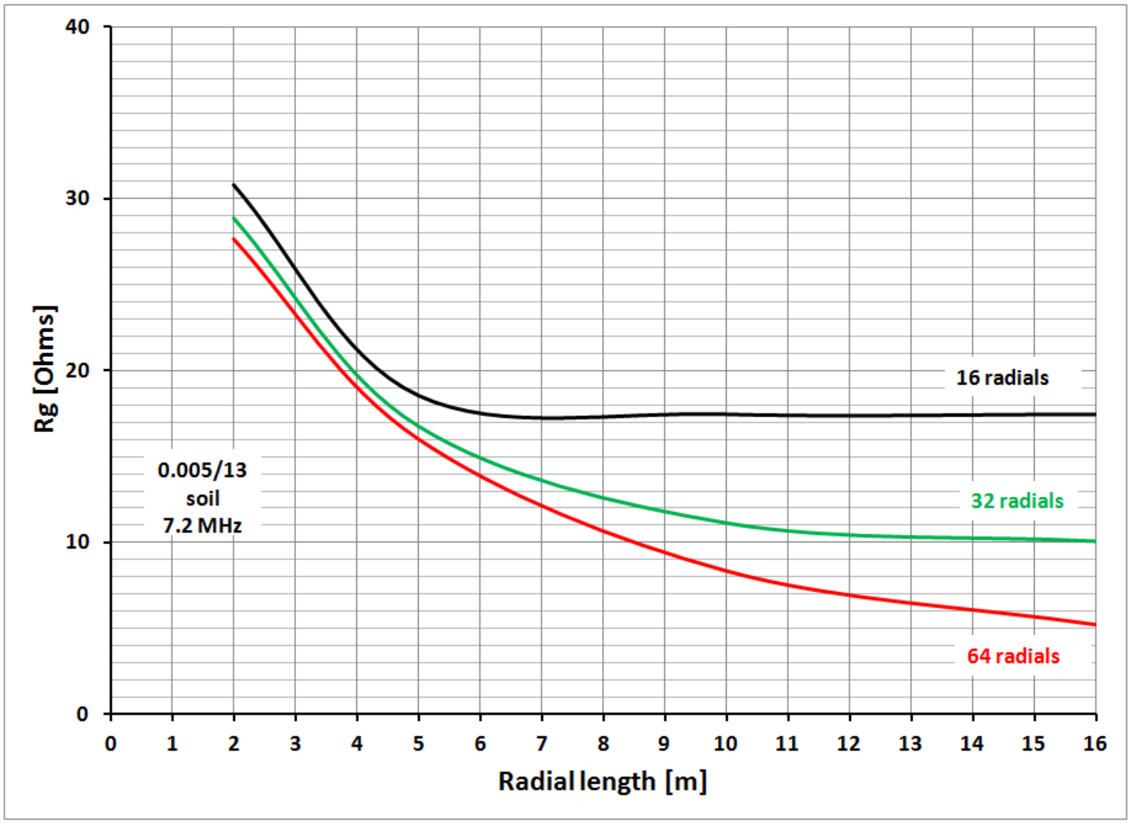

Figure 15 - Rg as a function of radial length and number.

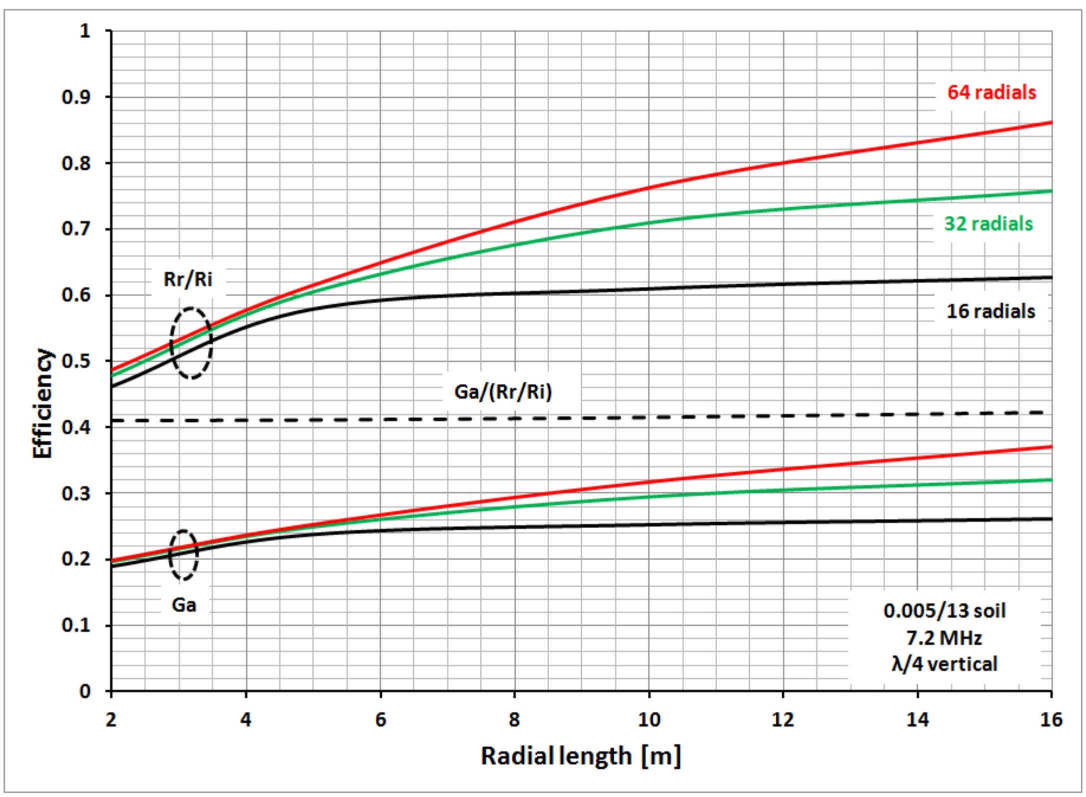

Figure 16 - Efficiency as a function of radial length and number.

For a given model, NEC will give us Ri, Ga and the field data from which we can determine Rr using the Poynting vector and a spreadsheet. With this information we can have some fun! Rr**/**Ri is the radiation efficiency including only the ground losses within the radius of integration which in this case is ≈λ/2. Ga is the radiation efficiency including all the losses, near and far-field. The ratio Ga/(Rr/Ri) gives us the loss in the far-field separate from the near-field losses. Figure 16 graphs all three, Ga, Rr**/**Ri and Ga**/(**Rr**/**Ri**)** with various numbers of radials over average ground. Note that the far-field loss is almost independent of the radial number or radial lengths, which is what you would expect because we haven't changed anything in the far-field as we modified the radials. In fact any bumps or anomalies in that graph would indicate a screw-up in the calculations! It serves as a much needed cross check on the calculations.

After seeing figure 16 Steve Stearns, K6OIK, suggested adding a graph of (Ri/Rg)-Ga which is the ground wave radiation efficiency. This is shown in figure 17.

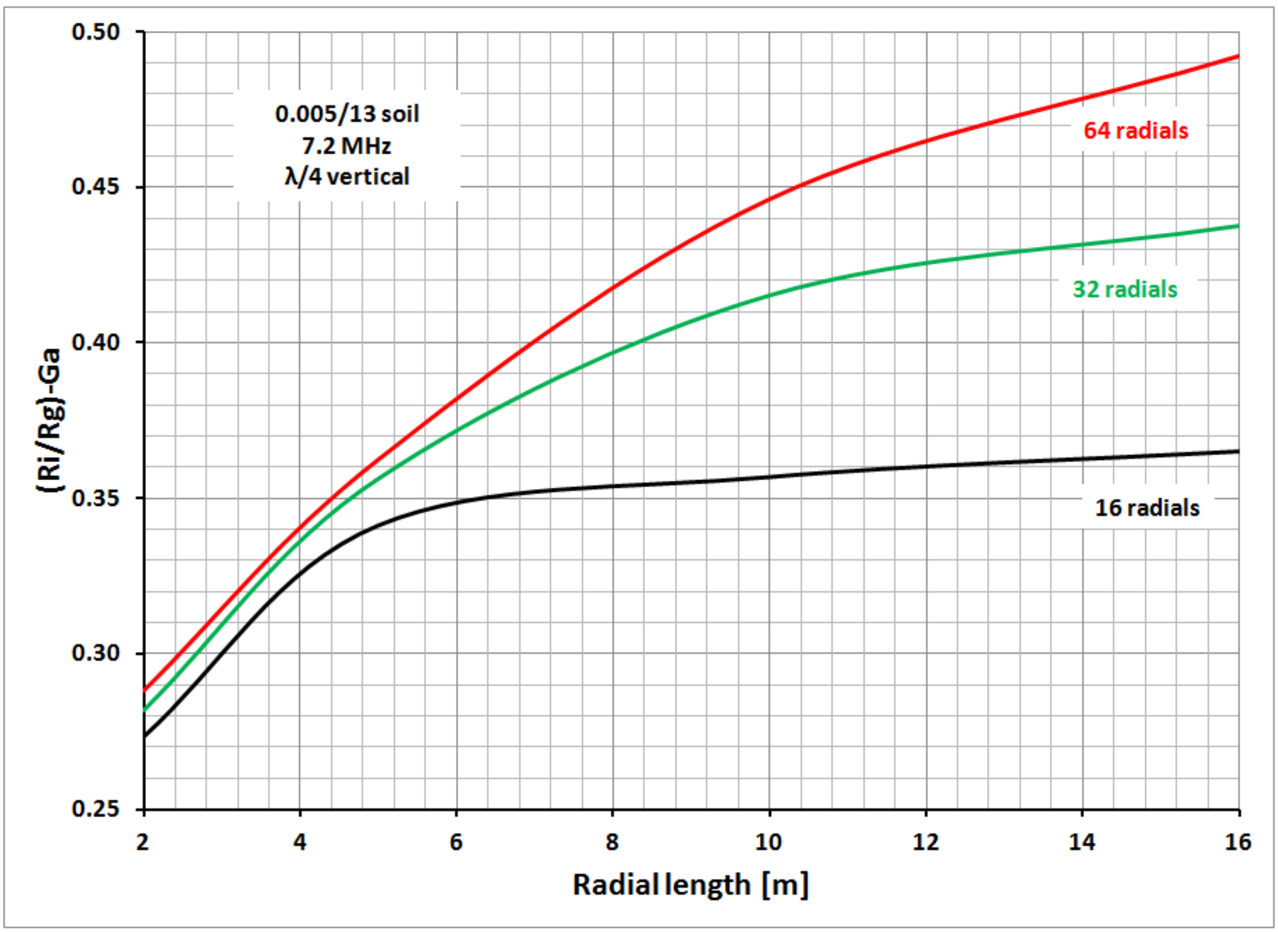

Figure 17 - Ground wave radiation efficiency (Ri/Rg)-Ga.

By repeating the calculations for a λ/4 vertical at 1.8 MHz we can compare the results to expose the effect of frequency on Ri, Rr and Rg for the same type of antenna. An example is given in figure 18. The solid lines are for 1.8 MHz and the dashed lines 7.2 MHz.

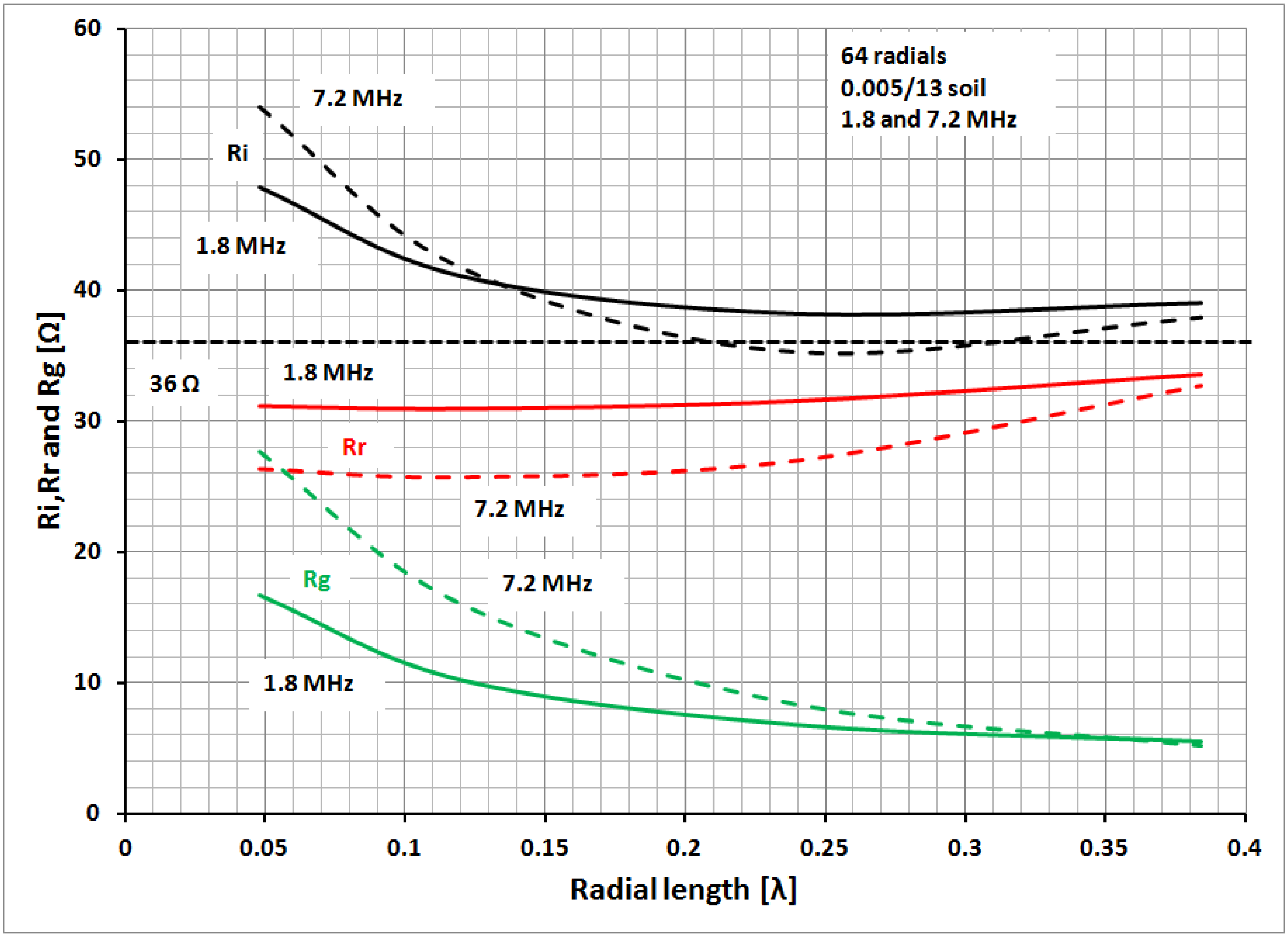

Figure 18 - Ri, Rr and Rg for a λ/4 vertical with 64 radials at 1.8 and 7.2 MHz.

What we see is that even though both antennas are λ/4, with the same length radials (in λ) and the same soil characteristic, the values for Ri, Rr and Rg are substantially different. At 1.8 MHz Rr is much closer to 36Ω. Using λ/4 radials at 7.2 MHz and integrating the radiated power, Rg≈8Ω. However, if you subtracted the Ri value given by NEC from 36Ω you would think Rg was essentially zero! At 1.8 MHz Rg=36-Ri≈2Ω, which seems reasonable. However, the power integration for the 1.8 MHz vertical gives Rg≈6Ω which means the efficiency is lower than we thought. As the soil conductivity (σ) increases, the values for Rr move closer to 36Ω. If we lower the frequency to the lower broadcast band (say 600 kHz) using a λ/4 vertical with 120 0.4λ radials, Rr will be very close to 36Ω. This is a frequency range where a great deal of profession work has been done which might explain why the discrepancy between estimated and actual Rg and Rr went unnoticed. The difference would be very small, easily within the range of experimental error!

A small 630m vertical

On 630m (472-479 kHz), where λ≈2000', any practical amateur antenna is very likely to be small in terms of wavelength. Figure 19 shows an example of a short top-loaded vertical for 630m. The vertical is 15.24m high (50', 0.024λ) with 7.62m (25', 0.012λ) radial arms in the hat. The usual practice for very short verticals is to have a dense ground system which extends some distance beyond the edge of the top-hat and/or a bit longer than the height of the vertical. Two cases were modeled: 64 and 128 radials, all 18m long.

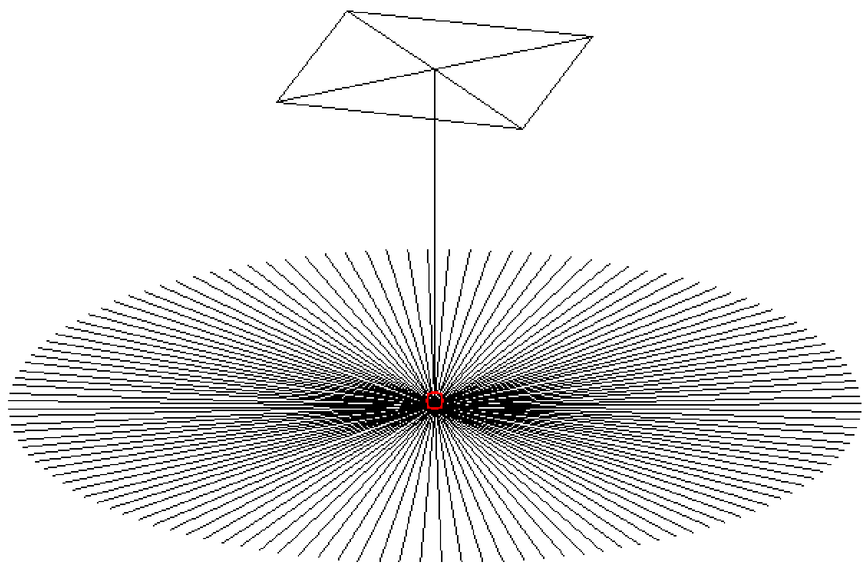

Figure 19 - 630m antenna example.

The results calculated from the NEC field data are given in table 2. Over perfect ground Rr=0.7Ω

Table 2A - 630m vertical 64 radials, integration radius =100m.

soil	Ri [Ω]	Rr [Ω]	Rg [Ω]	Rr**/**Ri	**0.7/**Ri	Ga
0.001/5	5.50	1.01	4.49	0.18	0.13	0.060
0.005/13	2.01	0.844	1.17	0.42	0.34	0.232
0.03/20	1.09	0.76	0.32	0.70	0.63	0.533
perfect	0.69	0.69	0	1.00	1.00	1

Table 2B - 630m vertical 128 radials, integration radius =100m.

soil	Ri [Ω]	Rr [Ω]	Rg [Ω]	Rr**/**Ri	**0.7/**Ri	Ga
0.001/5	4.90	1.009	3.895	0.21	0.14	0.067
0.005/13	1.883	0.843	1.04	0.45	0.37	0.247
0.03/20	1.033	0.78	0.253	0.76	0.67	0.561
perfect	0.69	0.69	0	1.00	1.00	1.00

For this antenna with real soils Rr is somewhat higher than the perfect ground case and converges on the perfect ground case as the soil conductivity improves. In this example using the perfect ground value for Rr yields an efficiency somewhat lower than real soil as shown in the 0.7**/**Ri column, but the difference is not very large. We should also keep in mind, as shown in appendix C, the computed values for Rr depend on the integration radius which is somewhat arbitrary. If I had used a slightly larger radius the Rr values would have been a bit lower, i.e. closer to the ideal ground value.

Summary

For a lossless antenna in a lossless environment, the calculation of radiation resistance is very straight forward: integrate the power density over a hypothetical surface enclosing the antenna. The net power outflow divided by the square of the rms current at the feedpoint gives Rr. We can extend this technique to antennas in a lossy environment by using the field values obtained from NEC modeling and a spreadsheet.

At HF, values for Rr over real soils appear to be significantly lower than the values for the same antennas over perfect ground, at least in the case of λ/4 and λ/2 verticals! However, for short verticals at MF, the real-ground Rr appears to be close to the ideal value depending on the details of the soil and the ground system. It's my opinion that calculating Pr and efficiency using the perfect ground value for Rr is a reasonable approximation for the verticals likely to be used by amateurs at 630m. However, that calculation will not be correct for an HF vertical over real soil.

Acknowledgements

I want to express my appreciation to Steve Stearns, K6OIK, for his very helpful review of this article. He put in a lot of effort and I've incorporated many of his suggestions in the main article and in the appendices. I also appreciate the comments from Dean Straw, N6BV, and Al Christman, K3LC. All of the modeling employed a prototype version of Roy Lewallen's EZNEC Pro/4[3] that implements NEC 4.2 and Dan MaGuire's (AC6LA) AutoEZ[6] which is an EXCEL spreadsheet which interacts with

EZNEC to greatly expand the modeling options. Without these wonderful tools this study would not have been practical and I strongly recommend both programs.

References

[1] Frederick Terman, *Radio Engineers' Handbook*, McGraw-Hill, 1943

[2] John Kraus, Antennas, McGraw-Hill, 1988, second edition

[3] EZNEC pro/4 by Roy Lewallen, www.w7el.com

[4] King and Smith, Antennas in Matter, MIT press 1981, page 399, section 6.8

[5] J. Wait, R. Collin and F. Zucker, *Antenna Theory*, Chap 23, Inter-University Electronics Series (New York: McGraw-Hill, 1969), Vol 7, pp 414-424.

[6] AutoEZ by Dan McGuire, AC6LA, http://ac6la.com/autoez.html