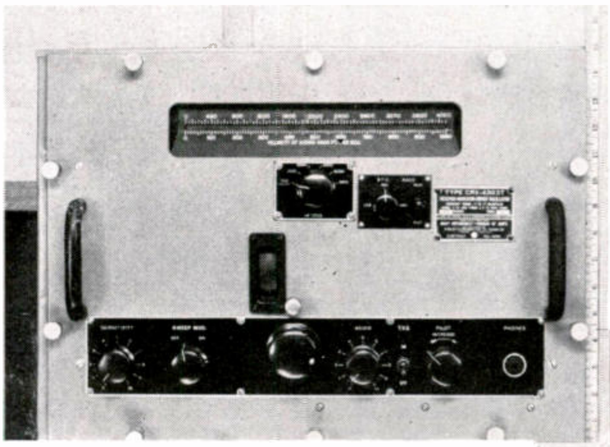
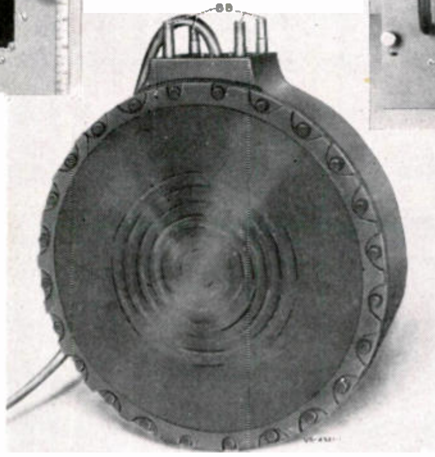
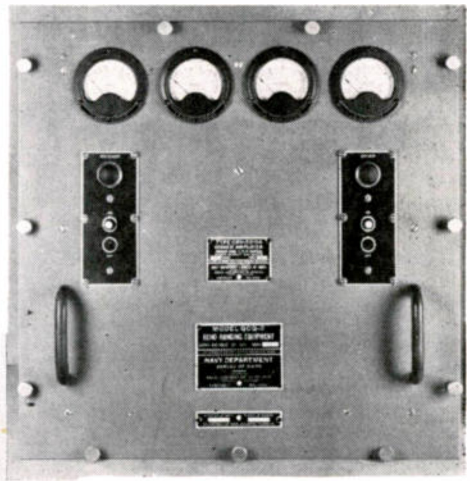
PROCEEDINGS OF THE I·R·E

AND

WAVES AND ELECTRONS







RCA-USN

SONAR—UNDERWATER SOUND ECHO-RANGING-LISTENING SYSTEM

The driver-oscillator portion of the assembly (upper left) excites the projector (center) through the driver amplifier (upper right). The tell-tale echo, picked up by the projector, is fed into the receiver-indicator portion of the unit at upper left.

September
1946

Volume 34 Number 9

The Institute of Radio Engineers

where

$$a = \frac{1}{(R+R_0(1+G_mZ_L))c}$$

$$-E_0/E_1 = G_mZ_L\left(1-e^{-at}\left(\frac{R_0(1+G_mZ_L)}{R+R_0(1+G_mZ_L)}\right)\right) \quad (4)$$

which is a wave of the type shown in Fig. 6. It rises abruptly and then follows an exponential wave in increasing to its final magnitude. This is the type of wave required to drive a linear current through a combination of resistance and inductance in series if the time constant $(R+R_0(1+G_mZ_L))c$ is long, compared with the required length of sweep so that the exponential rise may be approximated by a linear rise. To show this,

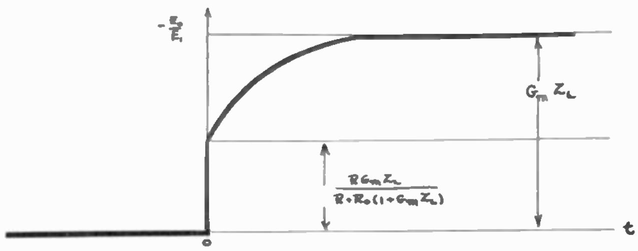


Fig. 6—Sweep-circuit output-voltage wave form.

consider a load consisting of R_1 and L in series, as shown in Fig. 7. The required condition is that $i = K/p$ where

$$K = \frac{\text{the total deflection current}}{\text{the total sweep time}}$$

$$E_0 = i(R_1 + Lp)$$

$$= KL + KR_1/p.$$

The associated time function is $E_0 = KL + KR_1t$ which should match (4) (approximating e^x by $1+x$).

From (4)

$$-E_0/E = G_mZ_L\left(1 - \frac{R_0(1+G_mZ_L)}{R+R_0(1+G_mZ_L)}\cdot\left(1-\frac{t}{(R+R_0(1+G_mZ_L))c}\right)\right)$$

$$= G_mZ_L\left[\frac{R}{R+R_0(1+G_mZ_L)} + t\frac{R_0(1+G_mZ_L)}{[R+R_0(1+G_mZ_L]^2c}\right].$$

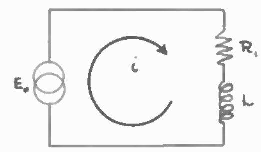


Fig. 7—Simple equivalent circuit of a magnetic deflection yoke.

Thus, the conditions that must be met by the values of L and R_1 of the magnetic-sweep inductor are

for $$\frac{1}{2[R+R_0(1+G_mZ_L)]^2c^2}t^2 < 0.1$$

(neglected term in the series expansion)

$$L = \frac{RA_0}{K[R+R_0(1+A_0)]}$$

$$R_1 = \frac{A_0R_0(1+A_0)}{K[R+R_0(1+A_0)]^2c}$$

or

$$L/R_1 = Rc\frac{R+R_0(1+A_0)}{R_0(1+A_0)}$$

where $A_0 = G_mZ_L$ = gain of the amplifier without feedback.

Special Aspects of Balanced Shielded Loops*

L. L. LIBBY†, SENIOR MEMBER, I.R.E.

Summary—**The theory of operation of the balanced shielded loop antenna is reviewed. A method of analysis of this type of antenna is described, wherein transmission-line principles are utilized to account for the distributed nature of the loop constants for loops whose perimeters are of the order of one-quarter wavelength. It is shown that the loop conductor within the shield may be treated as a coaxial transmission line having uniformly distributed constants, and that the outer surface of the shield may be treated as a balanced two-conductor transmission line having nonuniform constants. A method is described whereby the relatively cumbersome equations of the latter type of transmission line may bé avoided by the use of an "equivalent" line having uniform characteristic impedance. A sample calculation is included to illustrate the utility of this method of analysis.**

* Decimal classification: R125.3. Original manuscript received by the Institute, September 17, 1945; revised manuscript received, November 20, 1945.

† Federal Telecommunication Laboratories, Inc., New York, N. Y.

Introduction

The BALANCED shielded loop antenna is widely used in specialized types of radio equipments, e.g., direction finders, homing devices, etc. Its behavior at low frequencies is generally understood and has been covered quite completely in numerous texts on radio and communications. However, the high-frequency behavior of the loop requires further analysis, some of the features of which will be covered in this paper.

The basic principles underlying the operation of this type of antenna are fundamentally the same as those for any other type of antenna. This means that the loop antenna operates so as to satisfy Maxwell's equations at each and every infinitesimal point in space, whether this be a point on the loop shield surface, within the shield conducting material, or anywhere in the dielectric medium surrounding the loop antenna. Strictly speaking, then, the complete electromagnetic field, including all retardation effects, must be considered in analyzing the behavior of the loop antenna at high frequencies. However, it is often possible to take advantage of the analytical simplifications afforded by the use of conventional circuit theory, as will be shown.

In the discussion which follows the analysis will be restricted to the case of a single-turn, balanced, shielded loop wherein an inner conductor is positioned within a

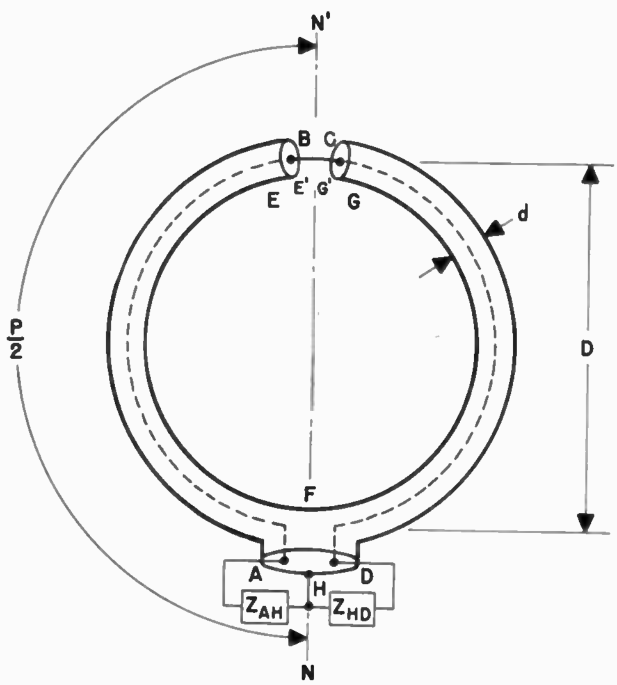


Fig. 1—Single-turn balanced shielded loop antenna.

shielding tube of highly conductive nonferrous material such as copper, aluminum, or brass. The conductivity is considered great enough so that the so-called "depth of penetration" of current and field is less than 10 per cent of the wall thickness of the tubing, thus ruling out any interaction between current on the outside of the shield and current on the inside of the shield. This restriction represents the usual case for shielded loops. A further restriction is made in that the half-perimeter ($P/2$) of the loop shield is not to exceed a length equal to one fourth of the free-space wavelength (i.e., 90 electrical degrees) of the highest frequency under consideration, thus ruling out cases where the loop-shield current may undergo a reversal of phase and hence complicate the analysis. Such a loop antenna is shown diagrammatically in Fig. 1, wherein the inner conductor $ABCD$ is contained within the outer shield EFG. Although this loop is pictured as being circular, the analysis applies equally to other commonly encountered shapes such as square, diamond, and rectangular with long axis vertical. The rectangular loop with long axis horizontal tends to act like a folded-dipole antenna, and, therefore, will not be considered in this discussion.

In accordance with the theory of symmetrical balanced circuits, the vertical axis $N-N'$ of the loop is the line of intersection between a virtual infinite equipotential plane and the plane of the loop. This virtual infinite plane is, of course, perpendicular to the plane of the loop. Thus, the balanced loop behaves the same as any other balanced or "push-pull" system, and it is possible to consider any point whose physical position coincides with the virtual equipotential plane to be at reference ground potential. (An implication contained in the above statement is that the axes of physical symmetry and electrical symmetry of the loop are the same.) Therefore, points F and H and the point on the inner conductor midway between points B and C may all be considered to be at reference ground potential. The half-perimeter of the loop is represented by $P/2$, this distance being measured along the center line of the shield tubing.

Basic Principles of Operation

The shielded loop receives energy from a vertically polarized, horizontally propagated electromagnetic wave by the following process:

The propagated field induces electromotive forces on the outside surface of the shield, along each of its legs, but induces none on the inside surface of the shield nor on the inner conductor, since the depth of penetration of the field is much less than the thickness of the shield material.[1] This fact, incidentally, permits us to treat phenomena on the outside surface of the shield independently from phenomena on the inside surface of the shield. Thus, for example, points E and G on the edge of the loop shield (Fig. 1) may be considered to be associated only with currents and impedances on the outside

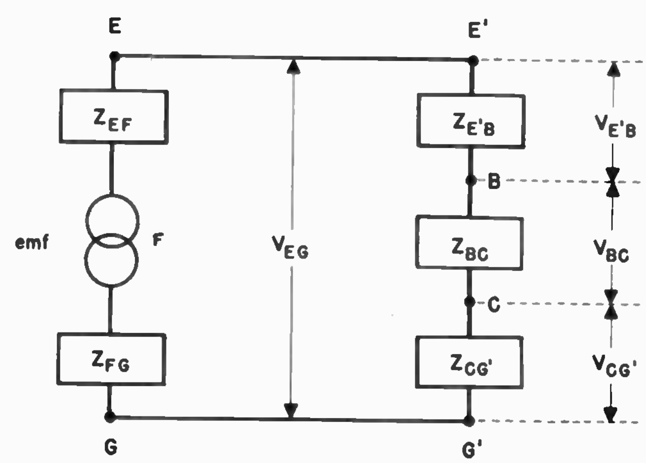


Fig. 2—Equivalent circuit of shield and gap impedances.

surface of the loop shield, whereas points E' and G' (actually the same points as E and G) may be considered to be associated only with currents and impedances on the inside surface of the loop shield. It should be noted that this inside surface of the shield and the inner

[1] S. A. Schelkunoff, "Electromagnetic Waves," D. Van Nostrand Company, Inc., New York, N. Y., 1943, Chapter 4, page 89.

conductor of the loop form a coaxial transmission line, a fact which will be made use of later in the analysis.

The electromotive forces induced along the outside surface of the loop-shield legs cause current to flow thereon and produce a resultant voltage V_{EG} across the shield gap EG (or $E'G'$). This gap voltage is thus impressed across the points $E'B$, BC, and CG' in series, so that the resultant voltages appearing across these individual points ($E'B$, BC, and CG') will be proportional to the impedances existing between them. This may be understood by referring to Fig. 2, which shows the equivalent circuit of the loop-shield and gap impedances. This method of analysis has been presented in a previous paper.[2]

Evaluation of Loop Impedances

It is obvious that for small gaps the impedance between points B and C is negligibly small, since it is composed of the inductive reactance of but a very short length of connecting wire. As a consequence, the loop-shield-gap voltage can be considered as being impressed only across impedances $Z_{E'B}$ and $Z_{CG'}$ in series, resulting in voltages $V_{E'B}$ and $V_{CG'}$. These voltages will be equal if the corresponding impedances are equal. The impedances will be equal if the two coaxial transmission lines, formed by the two legs of the loop shield surrounding the inner conductor, are equal in Z_0, in electrical length, and in terminating impedances Z_{AH} and Z_{HD}.

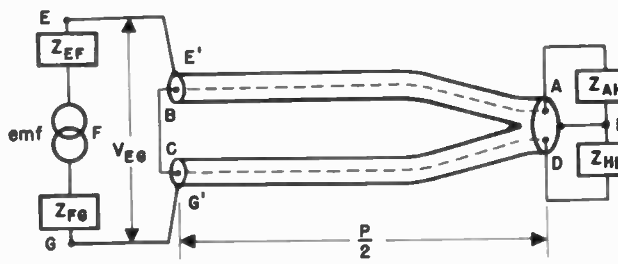


Fig. 3—Modified equivalent circuit showing coaxial lines.

Thus, referring to Fig. 3, we can see that the evaluation of the impedances $Z_{E'B}$ and $Z_{CG'}$ is resolved into the comparatively simple problem of solving the well-known equation for the input impedance of a transmission line of known termination. One convenient form of this is

$$Z_{IN} = Z_0 \frac{1 + \rho\underline{/-2\theta}}{1 - \rho\underline{/-2\theta}} \qquad (1)$$

where Z_0 = the characteristic impedance of the coaxial line formed by the loop conductor within the shield,

θ = electrical length of the line in degrees, and

ρ = the reflection factor.

The reflection factor ρ is given in turn by the equation

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0}, \qquad (2)$$

where Z_L is the terminating impedance of the line, represented by the impedances Z_{AH} and Z_{HD} in the illustration.

As a further simplification of the analysis, it is convenient to make use of the fact that the magnitude and phase angle of the impedance of any two-terminal network is unaffected by the order of connection of the terminals of this network into any circuit. This allows us to substitute Fig. 4 for Fig. 3, wherein terminals B and C of the coaxial transmission lines have been interchanged respectively with terminals E' and G'. This interchanging of terminals, applying as it does only to

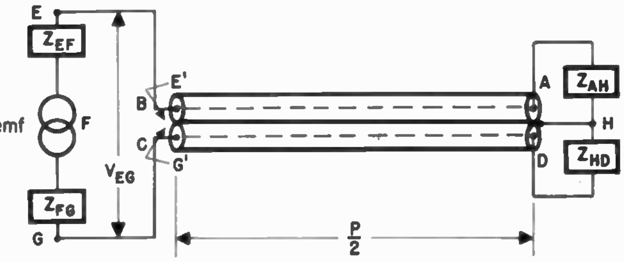


Fig. 4—Simplification of transmission-line connections.

phenomena associated with the inner surface of the loop shield, in no way affects the action of the outer surface of the shield with respect to the impedances Z_{EF} and Z_{FG}. It allows us to treat the two coaxial sections as two halves of a conventional balanced transmission-line system in which the shields are connected together, rather than separated.

The evaluation of the impedances Z_{EF} and Z_{FG} of the outside surface of the loop shield is comparatively simple at the lower radio frequencies (i.e., at frequencies where $P/2$ is less than 10 electrical degrees) since the current along the length of the shield is then substantially constant in amplitude and phase. This allows us to calculate the inductance of the outer shield, and also the radiation resistance, using standard formulas. The inductance for the case of a circular loop carrying current of constant amplitude and phase is given to a close approximation by the expression (referring to Fig. 1)

$$L = 0.01595D\left(2.303 \log_{10} \frac{8D}{d} - 2\right) \text{ microhenry}, \qquad (3)$$

where D and d are in inches, and the radiation resistance for this same case is given by the approximate expression

$$R = 31{,}000 \frac{A^2}{\lambda^4} \text{ ohms}, \qquad (4)$$

where A is the area enclosed by the loop-shield centerline in square meters and λ is the wave length in meters.

Examination of (4) shows that the radiation resistances of the loop shields considered above are of the order of 0.002 ohm or less, and as such are negligible compared to the corresponding inductive reactances. The impedances Z_{EF} and Z_{FG} are thus substantially pure inductive reactances at low radio frequencies, and

[2] N. Marchand, "Complex transmission line network analysis," *Elec. Comm.*, vol. 22, pp. 124–129; no. 2, 1944.

the equivalent circuit in Fig. 4 becomes that shown in Fig. 5.

The value of L referred to in this figure is that calculated from (3), and the electromotive force of the generator shown is obtained from the expression

$$\text{electromotive force} = h_e \mathcal{E} \text{ volts,} \tag{5}$$

where $\mathcal{E}$ is the field strength of the received wave in volts per meter, and h_e is the so-called effective height of the

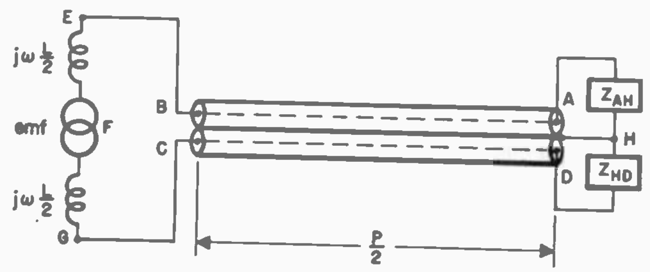


Fig. 5—Low-frequency condition corresponding to Fig. 4.

loop shield in meters, it being assumed above that the horizontal direction of propagation of the wave is in the plane of the loop.

The effective height h_e is given in turn by the following equation for a single-turn loop,

$$h_e = 2\pi A/\lambda, \tag{6}$$

where A and λ are the same as for (4).

For the higher radio frequencies the evaluation of the impedances Z_{BF} and Z_{FG} and of the total induced electromotive force becomes a somewhat more difficult problem than for the low-frequency case, since the current distribution along the shield can no longer be considered uniform. The current distribution is closely sinusoidal, having a maximum value at the base of the loop shield and dropping off in each leg as the gap position is approached. As a consequence, the loop shield tends to behave like a section of balanced transmission line, but not of uniform characteristic impedance. This nonuniformity of characteristic impedance is obviously a result of the fact that the current in each element of length of one leg of the loop shield is not fixed in distance and/or direction with respect to the oppositely flowing current element in the other leg of the loop shield. Because of this, relations which apply for uniform transmission lines do not apply in their regular sense for voltages, currents, and impedances involved on the outside surface of the loop shield. A quantitative analysis of the induced electromotive-force relationships is beyond the scope of this paper; but an analysis of impedance values is given below which should aid in establishing a picture of what takes place.

Referring to Fig. 6, the loop shield may be considered to comprise a two-conductor balanced transmission line of nonuniform characteristic impedance wherein each leg of the loop shield is a conductor of this transmission line, and the spacing S_x between conductors is a function of the distance x from the gap, as is the angle ϕ_x between the conductors (and between their currents I_x and I_x') at this point. It is evident from the figure that the terminating impedance of this nonuniform transmission line is zero, since at the point where the distance x becomes equal to the diameter of the loop the two conductors join to form a short circuit, and the shield current I becomes a maximum. In attempting to obtain the solution for the input impedance of such a transmission line, one usually encounters some rather cumbersome mathematical expressions which tend to make this work laborious. As a consequence, the writer has developed an approximate expression, verified empirically for a number of different cases, which greatly simplifies the problem of evaluating the above impedances. This empirical relation may be obtained from the following considerations.

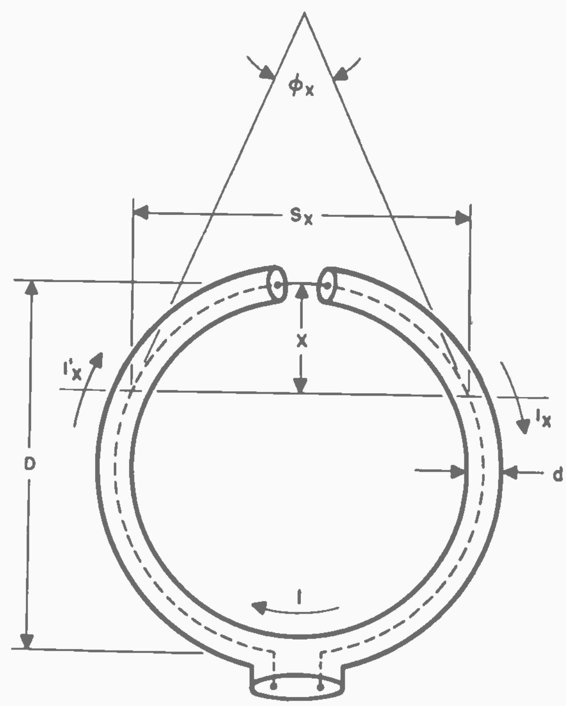


Fig. 6—Loop shield considered as a nonuniform transmission line.

The loop-shield configurations discussed above may be reduced to an equivalent uniform transmission-line section by postulating that:

(1) the length of this transmission line section be equal to the half-perimeter ($P/2$) of the loop shield;

(2) the conductors of this transmission line be of the same cross-sectional dimensions as the legs of the loop shield;

(3) the mean spacing between these conductors be such that the mean area of this equivalent transmission-line section be equal to the mean area of the loop shield; and

(4) the transmission-line section be perfectly short-circuited at its far end.

By applying these rules we get the transmission-line configuration shown in Fig. 7 as the equivalent of the loop-shield configuration of Fig. 6, and it is now possible to calculate the characteristic impedance of this transmission-line section by using the well-known relation (referring to Fig. 7)

$$Z_0 = 276 \log_{10} \frac{2S}{d}. \tag{7}$$

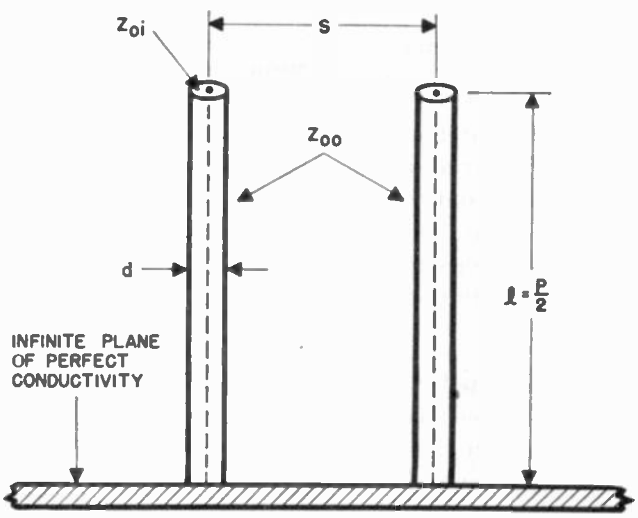


Fig. 7—Transmission-line section equivalent to the loop shield of Fig. 6.

This effective characteristic impedance of the loop shield we shall designate as Z_{00} to distinguish it from the characteristic impedance of the "inner" transmission line referred to in (1), which we shall now designate as Z_{0i}.

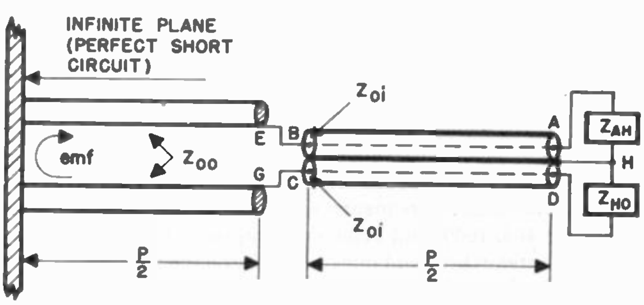


Fig. 8—High-frequency condition corresponding to Fig. 4 and incorporating the equivalent transmission-line section for the loop shield.

We may now draw the equivalent transmission-line network, Fig. 8, which is applicable to our problem for the high-frequency condition. This problem has now been reduced to the relatively simple case of a composite transmission-line system, the solution for the voltages, currents, and impedances of which may be obtained in the usual manner.

As a matter of note, the transmission-line network of Fig. 8 may be employed for the low-frequency solution also, since the relationships and equivalences established therein do not depend on frequency. Thus, calculating the inductance of the loop shield by means of the well-known equation for the input impedance of a section of short-circuited lossless transmission line,

$$Z_{IN} = jZ_0 \tan \theta, \tag{8}$$

we get for the inductance

$$L = \frac{Z_{IN}}{j\omega} = \frac{1}{\omega} Z_0 \tan \theta, \text{ henry} \tag{9}$$

and we find that this gives a value for the loop-shield inductance which is within a few per cent of that given by (3), thereby helping to verify the validity of the "equivalent transmission-line" method of analyzing loop-shield impedances.

Numerical Example

An example of how the above principles for evaluating the loop impedance components may be applied in finding the resonant frequency of a typical shielded-loop structure is given below.

Referring to Fig. 1, suppose that the following values are chosen:

mean loop diameter $D = 12$ inches,
outside diameter of shield tubing $d = 1$ inch,
inside diameter of shield tubing $d' = 0.9$ inch,
diameter of inner conductor $d'' = 0.064$ inch, and
loop load impedances $Z_{AH} = Z_{HD} = \infty$ (i.e., open circuit).

Solving for the spacing S of the equivalent transmission-line section (Fig. 7), we get

$$S = \frac{A}{P/2} = \frac{\pi(D/2)^2}{\pi(D/2)} = \frac{6^2}{6} = 6 \text{ inches}. \tag{10}$$

The characteristic impedance of this section is then

$$Z_{00} = 276 \log_{10} \frac{2S}{d} = 276 \log_{10} 12 = 298 \text{ ohms}. \tag{11}$$

The characteristic impedance of each inner coaxial line is

$$Z_{0i} = 138 \log_{10} \frac{d'}{d''} = 138 \log_{10} 14 = 158 \text{ ohms}. \tag{12}$$

The half-perimeter $P/2$ is given by

$$P/2 = \pi(D/2) = 6\pi = 18.9 \text{ inches} = 0.48 \text{ meter}, \tag{13}$$

so that the corresponding electrical angle in degrees is, (assuming air dielectric throughout),

$$\theta = \frac{(0.48)(360)}{\lambda} = \frac{173}{\lambda} \text{ degrees}. \tag{14}$$

Since both the outside transmission-line section and the inside transmission-line section are equal in electrical length for this case, we may write

$$\theta_0 = \theta_i = \theta. \tag{15}$$

The total electrical length of the transmission-line network, θ_T, would then be the sum of θ_0 and θ_i if the characteristic impedance Z_{00} were equal to twice the characteristic impedance Z_{0i}. Since such is not the case, we must obtain the equivalent electrical length θ_{eq} of the outside transmission line with respect to the inner transmission lines. This is obtained by equating the impedance to the left of BC (Fig. 8) in terms of Z_{00} to the same impedance in terms of twice Z_{0i}, whereby,

$$jZ_{00} \tan \theta_0 = 2jZ_{0i} \tan \theta_{eq}, \tag{16}$$

which then yields

$$\theta_{eq} = \arctan \frac{Z_{00} \tan \theta_0}{2Z_{0i}} = \arctan \left(\frac{298}{316} \tan \frac{173}{\lambda}\right), \tag{17}$$

and then θ_T may be obtained from

$$\theta_T = \theta_i + \theta_{eq} = \frac{173}{\lambda} + \arctan \left(0.944 \tan \frac{173}{\lambda}\right). \tag{18}$$

To obtain the lowest frequency at which this transmission-line network goes through resonance, i.e., the frequency at which the reactive component of the impedance across terminals A and D becomes zero, we set θ_T equal to 90 degrees and solve for λ. This is done by first transforming (18) to

$$\tan \theta_T - \tan \frac{173}{\lambda} = 0.944 \tan \frac{173}{\lambda} + 0.944 \tan \theta_T \left(\tan \frac{173}{\lambda}\right)^2. \tag{19}$$

Then, dividing through by $\tan \theta_T$ to get rid of some undesirable infinities, and setting θ_T equal to 90 degrees, this expression reduces to

$$\tan \frac{173}{\lambda} = \sqrt{\frac{1}{0.944}} = 1.030 \tag{20}$$

whereby

$$\lambda = \frac{173}{\arctan 1.030} = \frac{173}{45.85} = 3.78 \text{ meters}, \tag{21}$$

so that the resonant frequency is

$$f = \frac{300}{\lambda} = 79.4 \text{ megacycles}. \tag{22}$$

Actual measurements of the resonant frequencies of shielded-loop structures similar to the one calculated above indicate that the calculations give values accurate within 5 per cent. This again helps to verify the validity of the "equivalent-transmission-line" method of analyzing loop-shield impedances.

Conclusions

It is to be concluded that the balanced shielded loop antenna may be analyzed by the use of conventional transmission-line theory; and if the simplifications introduced in this paper are utilized, the amount of labor in making such an analysis is greatly reduced.

The method of analysis may be extended to include cases wherein certain compensating impedances are introduced between points B and C and across points E and G (Fig. 1) of the loop, it then being necessary merely to include these impedances at the appropriate points in Fig. 2.

Equalized Delay Lines*

HEINZ E. KALLMANN†, SENIOR MEMBER, I.R.E.

***Summary*—An improved design of signal delay lines is discussed in which phase distortion is held within the narrow limits required. The decrease of time delay at higher frequencies due to decrease of effective inductivity is compensated by a rise in effective capacitance due to distributed bridge capacities. In some high-impedance lines the natural coil capacitance will suffice for this compensation; in other cases a controlled amount of bridge capacitance is introduced by means of floating patches of metal foil along the coiled conductor, insulated from it, from each other, and from ground. Echoes, due to mismatch of the lines at high frequencies, may be suppressed by dividing the winding into sections, each too short to yield an echo component within the transmitted frequency range. The design of typical delay lines for 400, 1000, and 3000 ohms impedance is discussed and their delay, attenuation, and impedance characteristics are shown. A delay line with lumped iron-dust cores is described. A practical design is presented for the lumped-parameter low-pass filter $m=1.27$, as used for delay lines with very low impedance and for very high voltages.**

Brief Preface on the Effects of Amplitude and Phase Distortion

Delay lines are now widely used to delay steep-fronted pulses and other wide-band signals for periods of the order of one microsecond. Equalized delay lines are especially designed for low signal distortion; they are, for example, used in self-triggering oscilloscopes to delay, with negligible distortion, the signal front until the sweep is under way.

For fair transient response, both the amplitude and phase characteristics of a system must be fair over an adequate frequency range. Examples[1] of the relation between the amplitude and phase characteristics and the transient response are shown in Figs. 1A, 1B, and 1C, where various combinations of amplitude and phase characteristics are plotted on the left and the resulting distortions of a square pulse are shown on the right. A pulse may be considered as composed of two unit steps which are of opposite sign and follow each other with a

* Decimal classification R282.1×R117.11. Original manuscript received by the Institute, January 7, 1946; revised manuscript received, March 5, 1946. This paper is based on work done for the Office of Scientific Research and Development under contract OEMsr-262 with the Radiation Laboratory, Massachusetts Institute of Technology.

† Formerly, Radiation Laboratory, Massachusetts Institute of Technology; now, consulting engineer, New York, N. Y.

[1] H. E. Kallmann, R. E. Spencer, and C. P. Singer, "Transient response," Proc. I.R.E., vol. 33, pp. 169–195; March, 1945.