The Screened Loop Aerial*

A Theoretical and Experimental Investigation

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SUMMARY.—This paper describes a theoretical and experimental investigation of the screened loop type of receiving aerial used in direction-finding and field-strength measurement.

The theory of the screened loop is analysed and the simplifying assumptions made are indicated, and are shown to lead to a good first approximation for the value of pick-up for the screened loop. The pick-up is computed for the special cases of the screen gap short circuited, capacitively loaded, and tuned. It is shown that enhanced pick-up may be obtained by the suitable capacitive loading of the gap in preference to an increase of the number of loop turns, and the practical aspect of this system is discussed.

Confirmatory experiments are described, and the paper concludes with an indication of a suggested continuation of the investigation.

1. Introduction

THE use of an overhead earthed electrostatic screen consisting of several horizontal wires to overcome antenna effect in loop direction finders was suggested by Blatterman in 1919[1] who stated that electrically a more symmetrical system was thereby obtained.

The important development due to Barfield in 1923[2] was the use for this purpose of a screen surrounding the loop, consisting of a bundle of parallel wires or a metal tube, broken at one point so as not to form a closed conducting circuit. In modern versions the screening generally consists of a tube enclosing the loop (which also gives protection against the weather and mechanical damage) or, less commonly, of a vertical cage of wires surrounding it. An examination of previous literature on the use of the screened loop in direction finding and field strength measurement failed to reveal any exact quantitative account of its receptivity. In view of certain work in progress in the Radio Department it was decided to continue the investigation of the screened loop initiated by Barfield in the above-mentioned work.

2. Theoretical Analysis of the Screened Loop

(a) General Theory

In order to obtain a quantitative theory of the screened loop, it is necessary to make certain assumptions regarding the e.m.fs induced in the screen and loop, and the mutual actions of the currents they produce.

In the analysis developed here the following simplifying assumptions are made:

(1) That the screen and loop each have an induced e.m.f. independent of the other, and proportional to the area-turns and magnetic intensity perpendicular to the plane of the screen (or loop), the gap being sufficiently small not to affect the screen e.m.f.

(2) That the screen and loop are coupled inductively only, the coupling being expressed by the mutual inductance M.

(3) That the linear dimensions of the screen are small compared with the wavelength, and that the currents in screen and loop can be considered uniform.

This last assumption is later justified; if it is not made, the problem requires a treatment based on transmission line theory as has been recently applied to closed aerials by F. M. Colebrook.[3]

With the above assumptions, the equivalent circuit becomes as in Fig. 1. Z_1 and Z_2† are the total impedances around the screen and loop circuit, e_1 and e_2 the induced e.m.fs and i_1 and i_2 the corresponding instantaneous currents. The ratio of e.m.fs $\frac{e_2}{e_1}$ is denoted by n (assumed real) which

* MS. accepted by the Editor, June, 1939.

† The presence of the screen increases the self-capacitance and H.F. resistance of the loop due to its proximity. This effect is minimised by running the loop wire centrally in the screen.

tends to equality with the number of loop turns n_2 as the cross-section of the screen decreases. The Kirchhoff equations of p.d. for the two circuits are hence :

$$\text{screen} \quad e_1 = Z_1 i_1 + j\omega M i_2 \qquad \ldots \quad (1a)$$

$$\text{loop} \quad e_2 = n e_1 = Z_2 i_2 + j\omega M i_1 \qquad (1b)$$

where $\omega = 2\pi \times$ frequency. These equations

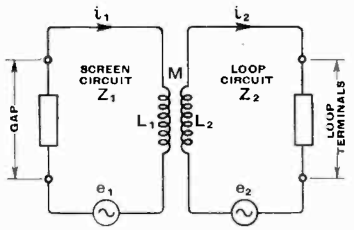

Fig. 1.—*Equivalent circuit of the screened loop.*

are soluble unless $Z_2 = j\omega n M$ and $Z_1 = \frac{j\omega M}{n}$ in which case they become identical and hence indeterminate.

The solutions for the currents are

$$\text{screen} \quad i_1 = e_1 \frac{Z_2 - j\omega n M}{\omega^2 M^2 + Z_1 Z_2} = \frac{e_1 - \frac{j\omega M}{Z_2} e_2}{Z_1 + \frac{\omega^2 M^2}{Z_2}} \qquad \ldots \quad \ldots \quad (2a)$$

$$\text{loop} \quad i_2 = e_1 \frac{n Z_1 - j\omega M}{\omega^2 M^2 + Z_1 Z_2} = \frac{e_2 - \frac{j\omega M}{Z_1} e_1}{Z_2 + \frac{\omega^2 M^2}{Z_1}} \qquad \ldots \quad \ldots \quad (2b)$$

In the second form of each, the numerator represents the effective e.m.f. and the denominator the effective impedance. In the numerator of the loop current, e_2 corresponds to the original field and $-\frac{j\omega M}{Z_1} e_1$ to the secondary field produced by the screen.

The mutual inductance M may be defined as the linkage between the loop and the magnetic flux produced by the unit current in the screen. Since each loop turn links with the screen flux, this linkage is equal to the (number of turns) $\times$ (flux produced by unit current in the screen i.e. L_1) and hence

$$M = n_2 L_1 \text{ or } k^2 L_2 = n_2^2 L_1 \qquad \ldots \quad (3)$$

where k is the coefficient of coupling between screen and loop of inductances L_1 and L_2 respectively. As already pointed out, $n(= e_2/e_1)$ will tend to equality with n_2 as the cross section of the screen decreases. In general, it will be approximately equal to n_2 and this equality will be assumed in all that follows. It is of interest to note that this equality must hold in the case of a closed perfectly conducting screen, for since there is no electric field within, zero resultant e.m.f. will act in the loop. Hence the e.m.f. e_2 in the loop due to the original field is exactly neutralised by the e.m.f. induced by the current $\frac{e_1}{j\omega L_1}$ in the screen.

$$\therefore \; e_2 - j\omega M \frac{e_1}{j\omega L_1} = 0 \text{ or } \frac{e_2}{e_1} = n = \frac{M}{L_1}$$

This argument cannot, however, be used to prove the equality of n_2 and n in general, since both M and L_1 will depend to some extent on the conductivity of the screen. Substituting nL_1 for M in (2b), the loop current becomes

$$i_2 = n e_1 \frac{(Z_1 - j\omega L_1)}{Z_1 Z_2 + \omega^2 M^2} \qquad \ldots \quad \ldots \quad (4)$$

It is now necessary to consider the two tuning conditions which are of practical interest : for maximum loop current, or maximum loop voltage. Let

$$Z_1 = R_1 + jX_1 \text{ and}$$
$$Z_2 = R_2 + jX_2$$

then taking C_2 as the tuning variable, the tuning condition for maximum current is given by the well-known relation

$$\frac{1}{\omega C_2} = \omega L_2 - \frac{\omega^2 M^2 X_1}{X_1^2 + R_1^2} \qquad \ldots \quad (5)$$

When tuned for maximum loop current the magnitude of the latter is found from equation (5) to be

$$I_2 = n e_1 \frac{\sqrt{R_1^2 + (X_1 - \omega L_1)^2}}{\left(R_2 + \frac{\omega^2 M^2 R_1}{R_1^2 + X_1^2}\right)\sqrt{R_1^2 + X_1^2}}$$

$$= I_{20} \frac{\sqrt{\frac{R_1^2 + (X - \omega L_1)^2}{R_1^2 + X_1^2}}}{1 + \frac{\omega^2 M^2 R_1}{R_2(R_1^2 + X_1^2)}} \qquad \ldots \quad (6)$$

where $I_{20} = \frac{n e_1}{R_2}$

is the value of the current at resonance for the unscreened loop.

The voltage tuning condition is very nearly identical with the current condition (5), except when X_1 has a value in the region of 0 or $\frac{\omega M^2}{L_2}$. Normally, and in the special cases discussed, the two conditions can be taken as identical, and the simpler current condition is therefore used.

(b) Normal Operation

When used normally, the screen gap is open-circuited and may in practice be completed with an insulator to protect the inner loop. Z_1 will then be a very large negative reactance. In this case the screen current will not be uniform along the length of the screen, and the analysis can only approximately represent the system. It indicates, however, that the terminal voltage of the loop should be very little affected by the screen, a result which is confirmed experimentally. It appears, moreover, that a non-uniform current distribution in the screen is not a necessary requirement as far as elimination of antenna effect is concerned, for, as is shown in the experimental section the efficacy of the screen in this respect is unimpaired by capacitive loading of the gap to an extent which results in an appreciable and substantially uniform screen current. It may be remarked that as e_1 and e_2 are strictly proportional to the magnetic flux linking the loop, it is reasonable to assume that the screening does not in any way affect the polarisation error of the loop or its susceptibility to local reradiation.*

(c) Three Special Cases

To compare the pick-up in the following cases, the p.d. V_2 across the loop terminals is evaluated and referred to the corresponding value V_{20} for the unscreened loop where

$$V_{20} = ne_1 \frac{\omega L_2}{R_2} = e_2 Q_2$$

* In field strength measuring apparatus which is calibrated by means of a series e.m.f., it appears to be implicitly assumed that the field acting on the loop is unaffected by the screen. An error from this source (although probably small in practice) is eliminated when the calibration is carried out on a known radiation field.

Case I—Gap Short-Circuited.—Here

$$Z_1 = R_1 + j\omega L_1 \text{ so } X_1 - \omega L_1 = 0$$

In all practical cases

$$\omega L_1 \gg R_1$$

The loop tuning condition (5) becomes

$$\frac{1}{\omega C_2} = \omega L_2(1 - k^2)$$

and the loop p.d. is then found, using equation (6), to be

$$V_2 = \frac{I_2}{\omega C_2} = ne_1 \frac{\omega L_2(1 - k^2)R_1}{\left(R_2 + \frac{M^2}{L_1^2}R_1\right)\omega L_1} \text{ very nearly}$$

$$= V_{20} \frac{1 - k^2}{Q_1 + k^2 Q_2}$$

where the magnifications are denoted by

$$Q_1 = \frac{\omega L_1}{R_1} \qquad Q_2 = \frac{\omega L_2}{R_2}$$

So the short-circuiting of the gap will reduce the pick-up to 1 per cent. or less of the value for the unscreened loop, and this was confirmed experimentally by Barfield[2].

In general the magnitude of the e.m.f. acting in the loop is given by

$$e = \left| e_2 - \frac{j\omega M}{Z_1} e_1 \right| = ne_1 \left| \frac{Z_1 - j\omega L_1}{Z_1} \right|$$

$$= ne_1 \sqrt{\frac{R_1^2 + (X_1 - \omega L_1)^2}{R_1^2 + X_1^2}} \quad \ldots \quad (7)$$

If the screen gap is short circuited

$$X_1 = \omega L_1$$

giving

$$e = \frac{ne_1}{\sqrt{1 + \left(\frac{\omega L_1}{R_1}\right)^2}} = e_2 \frac{R_1}{|Z_1|}$$

The "screening ratio" which measures the reduction of the e.m.f. induced in the loop by closing the screen is thus

$$\frac{e}{e_2} = \frac{e}{ne_1} = \left[1 + \left(\frac{\omega L_1}{R_1}\right)^2\right]^{-\frac{1}{2}} = \frac{1}{Q_1} \text{ very nearly} \quad \ldots \quad \ldots \quad (8)$$

As $R_1 \to 0$ this ratio tends to zero implying that as the closed screen becomes perfectly conducting the field within it is reduced to zero, which is in accordance with the theory of screening of alternating electric fields.

Case II—Gap Capacitively Loaded.—In this case

$$Z_1 = R_1 + jX_1$$

where X_1 is negative and $\gg R_1$

The loop tuning condition (5) becomes

$$\frac{1}{\omega C_2} = \omega L_2\left(1 - k^2\frac{\omega L_1}{X_1}\right)$$

and the resulting tuned loop p.d. is from equation (6)

$$V_2 = \frac{I_2}{\omega C_2} = V_{20}\frac{\left(1 - \frac{\omega L_1}{X_1}\right)\left(1 - k^2\frac{\omega L_1}{X_1}\right)}{1 + \frac{R_1}{R_2}\left(\frac{\omega M}{X_1}\right)^2} \quad .. \quad (9)$$

Hence V_2 can be greater than V_{20} since X_1 is negative in the case under discussion. Thus increased pick-up may be obtained by suitable capacitive loading of the gap.

It is shown in Appendix I that the optimum loading capacitance for maximum V_2 is given by

$$C_1 \text{ opt.} = C_{10}\frac{1 - k\sqrt{\frac{1 + k^2 q}{k^2 + q}}}{1 - k^2} \quad .. \quad (10)$$

where C_{10} is the capacitance to tune the screen to ω and

$$q = \frac{Q_1}{Q_2}.$$

The resulting maximum of the tuned loop p.d. is shown to be given by

$$V_2 \text{ max.} = V_{20} \cdot \tfrac{1}{2}\left[(1 + q) + \sqrt{\left(1 + \frac{q}{k^2}\right)(1 + qk^2)}\right] \quad .. \quad (11)$$

which is $\geqslant V_{20}\left(1 + \frac{Q_1}{Q_2}\right)$

But as

$$V_{20} = e_2 Q_2$$

the effective magnification of the loop is seen to be increased from Q_2 to at least $(Q_1 + Q_2)$ by optimum loading of the gap. Theoretical curves for the variation of $\frac{V_2}{V_{20}}$ with $\frac{C_1}{C_{10}}$ are given in Fig. 2, in which $k^2 = 0.5$, and q is given the values 0.5, 1.0, 1.5. The application of gap loading to increase the loop pick-up results in an increase of effective loop inductance (Appendix I) hence, for a given wavelength, a smaller tuning capacitance is required. In an unloaded screened loop the pick-up is proportional to the number of loop turns since the Q of the loop is approximately independent of the number of turns or wavelengths.

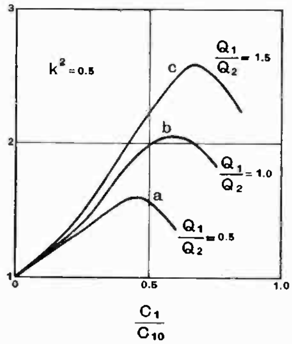

Fig. 2.—*Theoretical curves for the variation of pick-up with gap loading capacitance.*

If the pick-up is improved by either gap loading or increase of turns, the loop tuning capacitance must be decreased, the limit occurring when this capacitance is of the order of the loop self-capacitance. A general analysis given in Appendix II shows that under practical conditions gap loading will give a pick-up about 50 per cent. higher than increase of loop turns when both methods are taken to the limit set by loop self-capacitance, and this is confirmed by the test described in the experimental section.

The above considerations would apply to obtaining maximum pick-up from a given area of loop on a fixed wavelength (e.g. in aircraft or ship direction-finding).

Case III—Screen and Loop Tuned.—If the screen is tuned so that

$$X_1 = 0, \text{ that is } Z_1 = R_1,$$

the loop current will be maximum when

$$X_2 = 0, \text{ that is } Z_2 = R_2.$$

In this condition

$$\frac{V_2}{V_{20}} = \frac{\frac{\omega L_1}{R_1}}{1 + \frac{\omega^2 M^2}{R_1 R_2}} \quad \text{very nearly}$$

$$= \frac{Q_1}{1 + k^2 Q_1 Q_2}$$

$$= \frac{1}{k^2 Q_2} \quad \text{very nearly}$$

whence it is seen that the pick-up of the loop is considerably reduced. This occurs on account of the large resistance

$$\frac{\omega^2 M^2}{R_1} = \frac{\omega^2 L_1{}^2 n^2}{R_1}$$

injected by the tuned screen into the loop.

(*d*) *The Correlation of the Electric and Magnetic Viewpoints*

It is now interesting to examine the electric field conception of the injection of e.m.f. in the loop, treated in detail by Barfield[2]; namely, that when the loop is unscreened, the e.m.fs in the sides are combined vectorially to give the resultant e.m.f., while if the loop is screened the e.m.fs in the sides are reduced to very low values (to zero if the screen were perfectly conducting) but a potential difference exists across the screen gap producing an electric field which acts on the loop at the gap. From this viewpoint e.m.f. is induced into the loop almost wholly at the gap, and is proportional to the p.d. across the gap and the number of loop turns. It is seen that on dividing equation 1(b) by n:

$$e_1 - j\omega L_1 i_1 = \frac{Z_2 i_2}{n} \quad \ldots \quad \ldots \quad (12)$$

If R_1 = screen resistance, the screen impedance between the gap $= R_1 + j\omega L_1$. Therefore the gap p.d. V_g is given by

$$V_g = e_1 - (R_1 + j\omega L_1) i_1 - j\omega M i_2$$

$$= \frac{1}{n}(Z_2 - j\omega n M) i_2 - R_1 i_1$$

$$\therefore \quad i_2 = \frac{n(V_g + R_1 i_1)}{Z_2 - j\omega n M} \quad \ldots \quad \ldots \quad (13)$$

Here the term $R_1 i_1$ appears as a small addition to V_g arising from the imperfect conductivity of the screen. This equation suggests that the screen gap behaves to each loop turn as if it were a generator of open-circuit e.m.f. V_g and internal impedance $-j\omega M$.

3. Experiments

Experiments were carried out on three different loops of which particulars are given in Table I. The screened loops were all square in shape, and both the screen and the loop were of circular cross-section. The screen of the first two loops was of aluminium tubing, whilst the third was constructed from screened flexible cable. In all the subsequent measurements the accuracy is of the order of $\pm$ 5 per cent.

(*i*) *Test of the relation* $M = nL_1$.—In the analysis it is assumed that $\frac{e_2}{e_1} = \frac{M}{L_1} = n_2$ (the number of loop turns) and although this is obviously true at low frequencies where the currents are uniform it is desirable to show that the relation is still approximately valid at high frequencies even with a high impedance gap (i.e. non-uniform screen current).

(*a*) The constants M and L_1 were measured at audio-frequency ($\omega = 5 \times 10^3$) by a Campbell bridge and radio-frequency ($\omega = 4 \times 10^7$) by means of a Q meter. The ratio M/L_1 is tabulated in columns 3 and 4 of Table II.

(*b*) In this experiment a toroid encircling one limb of the screened loop was used to produce a known magnetic flux linking with it. The toroid is useful for such measurements on account of its low external field and, by keeping the toroid circuit away from resonance, the resistance and reactance it reflects into the loop are negligible. The screen was tuned by a condenser across the gap, and the loop open-circuited. The potential differences V_1 across the gap and V_2 across the loop were measured by means of valve voltmeters and their ratio found.

If the loop impedance is taken as infinite,

$$I_2 = 0, \text{ hence } V_2 = |e_2 - j\omega M i_1| \text{ but}$$

$$\omega M i_1 = \frac{\omega M}{R_1} e_1 \gg e_2$$

$$\therefore \quad V_2 = \omega M I_1 = \omega M \frac{V_1}{\omega L_1} = \frac{M}{L_1} V_1 \text{ or}$$

$$\frac{V_2}{V_1} = \frac{M}{L_1}$$

TABLE I

Loop	Screen o.d.	Loop o.d.	Loop turns	Side of screen	Gap width
	cms.	cms.		cms.	cms.
1	3.4	0.092	1	103	2.2
2	3.4	0.092	3	103	2.2
3	0.38	0.13	1	36	1.0

This experiment has the advantage that the quantities M and L_1 are simultaneously measured under identical conditions. The last column of Table II gives the ratio of the observed voltages for the three loops, and comparison of the tabulated values shows confirmation of the relation $M = nL_1$ within the accuracy of measurement. Hence it seems justifiable to neglect any capacitive coupling between loop and screen in the approximate theory of the screened loop (assumption 2).

TABLE II

Loop	Loop turns n_2	Audio frequency M/L_1	Radio frequency M/L_1	$\frac{V_2}{V_1}$
1	1	1.06	0.96	1.05
2	3	3.11	2.85	3.10
3	1	0.99	0.99	0.98

(*c*) It might be argued that, since in normal operation the screen is open-circuited at the gap, the values of $\frac{M}{L_1}$ measured by the two previous methods are not relevant for they involved the connection of a low impedance across the gap. To check this, a known H.F. current was sent through the loop (1) and the potential difference across the gap measured with a valve voltmeter having an input capacitance of only 10 $\mu\mu$F. The values of M measured at eight frequencies by this method gave a mean of 2.64 μH as compared with 2.73 μH obtained by the previous methods.

(*ii*) *Practical Test of Gap Loading*

(*a*) *Pick-up.* With a calibrated variable condenser across the gap the variation of the tuned loop p.d. with gap capacitance was measured, using, first, an e.m.f. injected by a toroid placed round one limb of the screened loop (No. 1), and secondly, a vertically polarised radiation field. The results are plotted in Fig. 3. The curves illustrating the results obtained by the two methods agree reasonably well with each other and with the theoretical curve, which is also shown in Fig. 3. These curves all apply to the frequency of 4.6 Mc/s. and the Q values were measured by means of a "Q meter" on this frequency, correction being applied for the self-capacitance of the screen and loop. This gave $Q_1 = 67$, $Q_2 = 102$, so $q = 0.66$; also $k^2 = 0.51$ as determined from measurements of the loop and screen self and mutual inductances.

As mentioned in the discussion of Case II the upper limit of the capacitive loading of the gap is set theoretically by the loop tuning. Loop No. 1 was used since it had the smallest self-capacitance (48 $\mu\mu$F) of the three and thus permitted the largest variation of gap loading. At the peak of the loop voltage

$$\frac{C_1}{C_{10}} = 0.5 \text{ app.}$$

the theoretical value (from equation 10) being 0.50. It can be shown (Appendix I) that at this gap loading the effective loop inductance is increased by a factor of 1.5.

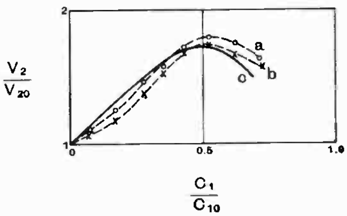

Fig. 3.—*Variation of pick-up with gap loading capacitance:*

(*a*) *measured, injection by toroid*

(*b*) *measured, injection by radiation*

(*c*) *theoretical*

It is seen from Fig. 3 that the pick-up factor has been increased by a factor of nearly 1.8 in this way. If, however, two turns had been used to double the pick-up factor, then the loop inductance would have been increased roughly fourfold and the loop self-capacitance roughly doubled so that their product would have been increased by a factor of about 8 which is certainly greater than 1.5 and would have made the loop untunable. Hence, with limitation due to loop self-capacitance, it is seen that screen loading is in this case advantageous compared with increase of loop turns.

(*b*) *Antenna Effect.*—Tests were made with a portable screened-loop direction-finder receiving a radiation field to ascertain whether gap loading and the consequent increase of current flowing in the screen impairs its screening properties, i.e. causes the appearance of antenna effect. The loop was arranged asymmetrically, one side being connected to the filaments and screening box of the receiver and the other to the grid of the H.F. amplifying valve. Used normally over a waveband of 25–50 m., the direction-finder showed no discrepancy greater than $\frac{1}{4}^{\circ}$ between the reciprocal bearings, while a swing of only 1° was necessary in taking the bearings. The reciprocal effect was measured for an unscreened loop of the same dimensions in place of the screened loop, and it was found to be of the order of 10°–20°, swings of 40°–80° being required.

Loading the gap with capacitance in steps up to twice the value needed to resonate with the screen showed that no additional antenna effect was introduced thereby. It should be noted that if antenna effect is initially present (due say to stray pick-up on leads or receiver) the reciprocal error will vary as the gap is loaded since the ratio $\dfrac{\text{wanted pick up}}{\text{unwanted pick up}}$ is varied.

(iii) Screen and Loop Tuned

Using the direction-finder previously mentioned, capacitances were successively placed across the gap, the loop retuned, and the signal strength noted. It was observed to fall to a very low value at screen resonance. With the loading condenser less than the resonant value the loop tuning condenser had to be reduced, whilst with a greater value it had to be increased relative to its normal value.

A check was made using toroidal injection, the loop being tuned and the loop p.d. measured. The screen was then tuned and the loop potential difference remeasured. Using loop No. 1 the reduction was found to be approximately $\frac{1}{35}$ which agrees with the theoretical value of

$$\frac{1}{k^2 Q_2} = \frac{1}{0.51 \times 70} = \frac{1}{36} \text{ (at 3 Mc/s.)}$$

(iv) Tests of the Effect of the Gap Condition

(*a*) The result found by Barfield[2] that the gap width is without detectable effect on the loop pick-up for value of $\frac{1}{2}$in. to the smallest obtainable, was confirmed for the range 22 mm. to 1 mm.

(*b*) An overlapping gap was constructed having the form shown in Fig. 4, resulting in effect in the introduction of a cylindrical condenser across the gap. The only effect to be expected is a slight increase of pick-up owing to the capacitive loading of the gap.

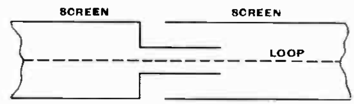

Fig. 4.—*Section through the "overlapping gap."*

Applying this type of gap to loop No. 1, it was found that an increase of 7 per cent. in the pick-up factor was obtained, which is exactly accounted for by the introduction of capacitance (25 $\mu\mu$F) across the gap.

4. Conclusion

A satisfactory simple theory of the behaviour of the screened loop in an electromagnetic field has been developed, on the basis of certain assumptions, which are stated at the beginning of the theoretical section. The further refinements of the theory which suggest themselves would require the investigation of:

1. The effect of the size and position of the screen gap upon the directional properties and pick-up of the screened loop.

2. The effect of the distributed self- and mutual-capacitance of screen and loop and the resulting non-uniformity of current.

5. Acknowledgments

The author is very indebted to his colleagues in the Radio Department of the National Physical Laboratory for their valuable guidance and interest in the investigation, and for advice on the preparation of the paper. The work was carried out as part of the programme of the Radio Research Board, and the paper is published by permission of the Department of Scientific and Industrial Research.

6. Appendices

Appendix I. (Analysis of Gap Loading).

Denoting $\frac{V_2}{V_{20}}$ by y and $\frac{\omega L_1}{-X_1}$ by x equation (9) becomes

$$y = \frac{(1 + x)(1 + k^2x)}{1 + n^2\dfrac{R_1}{R_2}x^2} = \frac{(1 + x)(1 + k^2x)}{1 + \dfrac{k^2}{q}x^2} \quad \ldots (9')$$

since

$$n^2\frac{R_1}{R_2} = k^2\frac{L_2}{L_1}\frac{R_1}{R_2} = k^2\frac{Q_2}{Q_1} = \frac{k^2}{q}$$

where $q = \dfrac{Q_1}{Q_2}$

If maximum current were required the factor $(1 + kx^2)$ would not occur and the optimum value of x is found to be $\sqrt{q}/k$ which corresponds to the usual resistance matching conditions. To determine the optimum value of x, $\dfrac{dy}{dx}$ is equated to zero, giving :

$$x^2 + 2x\frac{1 - q}{1 + k^2} - \frac{q}{k^2} = 0$$

The positive root of this quadratic, which is required for capacitive gap loading is

$$x_{opt.} = \frac{q - 1}{1 + k^2} + \sqrt{\left(\frac{q - 1}{1 + k^2}\right)^2 + \frac{q}{k^2}}$$

$$= \frac{(q - 1) + \dfrac{1}{k}\sqrt{(q + k^2)(1 + k^2q)}}{1 + k^2}$$

$$\doteqdot \frac{\sqrt{q}}{k} \text{ if } q \text{ is approximately unity } (Q_1 \doteqdot Q_2)$$

The value of gap capacitance C_1 corresponding to a value of x is found from

$$x = \frac{\omega L_1}{-X_1} = \frac{\omega L_1}{\dfrac{1}{\omega C_1} - \omega L_1}$$

giving

$$C_1 = \frac{x}{1 + x}\frac{1}{\omega^2L_1} = \frac{x}{1 + x}C_{10}$$

where $C_{10} = \dfrac{1}{\omega^2 L_1}$ is the capacitance which tunes the screen to ω.

Hence the range $x = 0$ to ∞ corresponds to $C = 0$ to C_{10}. Substituting the value of $x_{opt.}$, the optimum gap capacitance is obtained

$$C_{1opt.} = C_{10}\frac{1 - k\sqrt{\dfrac{1 + k^2q}{q + k^2}}}{1 - k^2} \text{ which is} < C_{10} \quad \ldots (10)$$

The effective inductance of the loop is then

$$L_2 - \frac{1}{\omega}\frac{\omega^2M^2}{X_1} = L_2(1 + k^2x) \doteqdot L_2(1 + k\sqrt{q})$$

The substitution of $x_{opt.}$ into the equation for y gives

$$y_{max.} = \frac{V_{2max.}}{V_{20}} = \tfrac{1}{2}\left[(1 + q) + \sqrt{\left(1 + \frac{q}{k^2}\right)(1 + qk^2)}\right] \quad \ldots \ldots (11)$$

or

$$\frac{V_{2max.}}{e_2} = \tfrac{1}{2}\left[(Q_2 + Q_1) + \sqrt{\left(Q_2 + \frac{Q_1}{k^2}\right)(Q_2 + k^2Q_1)}\right] \quad \ldots \ldots (11')$$

now

$$\left(1 + \frac{q}{k^2}\right)(1 + qk^2) = 1 + q\left(\frac{1}{k^2} + k^2\right) + q^2 \geqslant 1 + 2_q + q^2$$

$$\therefore\ y_{max.} \geqslant (1 + q)$$

In practice k will lie within the limits 0.4 to 0.7 depending upon the ratio $\dfrac{\text{loop diameter}}{\text{screen diameter}}$ upon which Q_1 and Q_2 will also depend, and so by the correct choice of the ratio it is possible to secure the best value of V_2 max. The analysis only holds when X_1^2 and $(X_1 - \omega L_1)^2 \gg R_1^2$, that is away from screen resonance $(X_1 = 0)$ or short-circuit $(X_1 = \omega L_1)$.

Appendix II

If in a given screen a loop of n_0 turns be wound, then the product (loop inductance $\times$ self capacitance) may be empirically represented by an m-power law of the number of turns ; that is

$$L_2C_{s2} \propto n_0^{\ m}$$

If the gap is capacitively loaded the loop inductance will be effectively multiplied by a factor $(1 + k^2x)$ (see Appendix I) and hence less turns (n) must be used to resonate with the loop self-capacitance at the same frequency for now

$$n^m(1 + k^2x) = n_0^{\ m}$$

or

$$\frac{n}{n_0} = (1 + k^2x)^{-\frac{1}{m}}$$

The pick-up of the loaded loop is increased by at least $(1 + q)$ so the ratio of pick-up is

$$> (1 + q)\frac{n}{n_0} = \frac{1 + q}{(1 + k^2x)^{1/m}} = \frac{1 + q}{(1 + k\sqrt{q})^{1/m}}$$

using the approximation $x = \dfrac{\sqrt{q}}{k}$

Hence gap loading will give the greater pick-up if

$$mq > k\sqrt{q} \text{ or } m\sqrt{q} > k.$$

In this analysis $L_2C_{s2} \propto n^m$ and the exponent of n representing practical cases probably lies between 2 and 3, since roughly $L_2 \propto n^2$ and $C_{s2} \propto n$. Typical values are $q = 1$, $k = 0.7$ for which this inequality is seen to hold, the pick-up ratio having the values

$$\frac{1 + a}{(1 + k\sqrt{q})}\ \tfrac{1}{3} = 1.67 \text{ for an } n^3 \text{ law}$$

whilst

$$\frac{1 + q}{(1 + k\sqrt{q})}\ \tfrac{1}{2} = 1.53 \text{ for an } n^2 \text{ law}$$

Hence on the above premises, it may be concluded that gap loading with fewer turns is better than using the maximum number of turns without gap loading.

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