Multitone signalling system employing quenched resonators for use on noisy radio-teleprinter circuits

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Synopsis

The paper describes a method of transmitting teleprinter signals over noisy circuits. One tone in the low a.f. range is allocated to each of the 32 characters in the teleprinter alphabet, each tone being transmitted for the duration of the relevant character. At the receiver, the tones are detected by a set of resonators, one of which integrates the entire energy in the transmitted character before it is quenched at the end of the character period. As the tones are orthogonally related, the energy in the resonators corresponding with the unwanted characters is zero at the end of the integration period. Absolute amplitude levels are avoided in the assessing of the integrated energy. The equipment is synchronous, but accepts an input in the form of standard fully perforated tape, and uses a standard start-stop teleprinter for print out.

The performance of the system is analysed mathematically, and the results of practical tests are quoted and compared with the published performance of other telegraph systems. It is shown that it is possible to transmit 100 words/min with less than 0.2% errors when the signal is 4dB below the noise level in a bandwidth of 470c/s. This is an improvement over other known methods, and in particular it is very much better than the performance of a highly skilled Morse operator using a manual system.

1 Introduction

In spite of the vastly increased use of teleprinters on many radio circuits, there remains a considerable number that continue to use Morse code sent at hand speed. Such circuits are usually those which must use low-power transmitters and simple aerials, e.g. ships, some military circuits, etc. On such circuits the amount of traffic is often low, but the degree of urgency high, and practice has shown that a better service is given by the manual Morse operator than is possible with existing radio-teleprinter methods, because of the skill of the operator in interpreting a noisy signal. The communication efficiency of such a system is low, and many of these circuits could be converted to teleprinter service by the application of a signalling system that could perform on noisy circuits at least as well as a trained operator. The system to be described was designed to meet this need.

Basically, the system is a method of a.f. multitone signalling applicable to most kinds of a.f. channels, detection being carried out by means of quenched resonant circuits. The quenched-resonator principle employed has been used elsewhere, e.g. Téléminprimeur Coquelot, predicted-wave signalling,1 and Kineplex,2 etc., but the method described was devised independently and contains a number of unique features of fundamental importance.

The system can be designed for any speed of transmission and is capable of being time-multiplexed (the present model has been developed from an earlier 15-channel t.d.m. version) or frequency-multiplexed, and apart from its use in point-to-point h.f. communication for which it was designed, its particular characteristics suggest applications in long-wave telegraphy, telemetry and satellite communication.

The existing equipment uses the frequency band 320-660 c/s, and operates at a signalling rate of 10 characters/sec. This represents 100 words/min and is equivalent to 75 bauds in the associated teleprinter channel. The sound of the signal is very distinctive and has led to the adoption of the name Piccolo.

One Piccolo terminal occupies a cabinet measuring 22 X 21 X 26 in, and can be switched, by means of a front panel control, to either the sending or the receiving condition. Two identical units are therefore needed for full duplex working, but, should one unit fail, simplex working with the remaining unit may be resorted to.

The performance of the terminal equipment in the face of random noise has been measured as being within 8dB of the theoretical limit specified by the Hartley-Shannon law, an improvement of some 15dB over conventional wide-deviation f.s.k. systems and 2dB better than the theoretical limit of any bit-by-bit decoding system. The system has also been shown to give a high degree of protection against impulse noise, interference and selective fading. In comparison tests with experienced Morse operators, the system registered less than 0.1% errors when operating at 100 words/min under conditions in which the Morse operator could barely detect the presence of a signal and could obtain no useful copy.

2 Circuit description

2.1 Fundamental principle

If a burst of tone at resonant frequency is applied to a resonant circuit with finite losses, the envelope of the response will increase exponentially to a finite limit, as shown in Fig. 1a. If the applied frequency is not equal to the resonant frequency, an oscillatory transient occurs before the system reaches its ultimate level, as indicated in Fig. 1b. The duration of this transient increases as the selectivity of the circuit increases, and is a limitation on the use of a tuned filter to extract information from a varying signal.

However, a particular case occurs if there are no losses in the resonant circuit. A suddenly applied tone at the resonant frequency will cause a response, the envelope of which increases linearly at a finite rate, as shown in Fig. 1c. A tone at any other frequency will cause a transient response, the envelope being of the 'modulus-sine' form shown in Fig. 1d. The zeros of this waveform occur at time intervals of 1/Δf, where Δf is the frequency difference between the applied tone


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The resonant frequency of the tuned circuit. These results are established mathematically in Appendix 9.1.

The basic system, shown schematically in Fig. 2a, contains a set of lossless tuned circuits, each of which differs in frequency from the next by a fixed amount \( f_0 \). If a tone burst at a frequency coinciding with that of one of the circuits is applied to the complete set, the response of the circuit correspondence to the input frequency will build up linearly.

The response of the two adjacent circuits will pass through zero at time \( t_0 \). The zeros in the next channels away on either side will occur at times \( t_0/2 \), \( 2t_0/2 \) and so on. Thus the envelopes of the responses form an orthogonal set; i.e. at the time \( t = t_0 \), all members of the set are zero except that corresponding with the input frequency, which is finite. Therefore, if the set of circuits is inspected at a time \( t_0 \) after applying the burst of tone, the circuit corresponding in frequency to that of the tone will be capable of producing an output, while the output of all the others will be zero.

### 2.2 Sending system and tape reader

Each of the 32 characters in the teleprinter alphabet is allocated an audio frequency in the band 330–650 c/s (A = 330 c/s, B = 340 c/s, C = 350 c/s, etc., letter shift = 630 c/s, blank = 640 c/s). These tones are produced by a set of 32 LC oscillators. The spacing of 10 c/s is derived directly from the signalling speed of 10 characters/sec, since, to satisfy the requirements of Section 2.1, the spacing must be the reciprocal of the character duration.

The input is in the form of standard, fully perforated, 5-unit teleprinter tape. This is read by a photoelectric tape reader that forms an integral part of the apparatus. This reader is a small sub-unit that, in its sending position, projects from the front of the unit. It is hinged to the front panel by its lower edge and may be stowed by lifting it into the unit when the equipment is used only to receive.

A master oscillator (see Fig. 2b) is used to drive a small synchronous hysteresis motor in the tape head, and this rotates an output shaft at exactly 10 c.p.s. This shaft is rotating continuously when the equipment is operating (whether sending or receiving), and constitutes the master time and phase reference of the whole system. It performs three functions:

(a) tape transport
(b) timing waveform generation (see Section 2.6)
(c) standby frequency generation (see Section 2.7).

The tape transport mechanism consists of a standard tape feed roller with ratchet wheel and detent mechanism, the ratchet being gathered by a pawl driven from an eccentric on the output shaft. The motion of the tape is therefore smooth and quiet, and wear is negligible. Means are provided for stopping the motion of the tape without affecting the rotation of the output shaft.

The holes in the tape are read by five silicon photodiodes mounted below the tape track and illuminated from above by a linear-filament lamp mounted on a hinged flap that may be raised for easy tape insertion. At an instant during the time the tape is stationary, the output of each photocell is examined by a strobing pulse, judged to be either 'mark' or 'space', and a binary-element store circuit is set accordingly.

An improved version of the tape head, now under development, uses a gallium arsenide semiconductor lamp. In this version, the strobing pulse is applied to the lamp itself, and each resulting photocell output pulse is applied to an amplitude discriminator which performs the mark or space judgment.

The 5-wire output of the five binary stores is applied via a 10 \( \times \) 32 diode matrix to the oscillators, so that all except one are held in a heavily damped condition, while the one selected by the photocell combination is shock-excited to a preset amplitude and maintained there by a biased negative-feedback circuit.

The output of the oscillator unit thus consists of a single tone that is changed in frequency every 100 ms according to the 5-unit combination read by the photocells. This output is amplitude-modulated to about 10% by a 10 c/s square wave (for synchronization purposes), amplified to the required output power, and delivered to the output socket.

### 2.3 Receiving system

In the receiving equipment (see Fig. 2a), the signal is applied to a set of 32 tuned circuits, each tuned to one of
the allocated frequencies. Each tuned circuit has positive feedback applied to it so that the resonant circuit losses are just counterbalanced. The result is a tuned circuit which is effectively lossless or of infinite Q-factor. Alternatively, the circuit may be considered as an oscillator in which the damping is increased to the point that the circuit will maintain oscillation at any level to which it is energized, but will not build up oscillations spontaneously.

The resonators are heavily damped until the beginning of one of the character tone bursts, when they are all released simultaneously. The subsequent behaviour of the circuits may be seen in Fig. 3, where the responses of the A and B resonators are shown to the input signal B-A-C-K.

The 32 resonator outputs are each detected separately, and the detected outputs connected to a voltage comparator circuit. At a time 100 ms after the beginning of the tone burst, this circuit compares the outputs of all 32 detectors simultaneously, selects the one with the highest amplitude, and generates a pulse on the corresponding output wire. It should be emphasized that the comparator circuit selects the highest amplitude irrespective of its absolute level, and, as the discrimination is better than 0.2 V for any level between 0.3 and 10 V, the circuit has a wide dynamic range which makes the system extremely effective against selective fading. Immediately after the comparator circuit has made its assessment, all the tuned circuits are rapidly quenched to zero and
It is arranged that the comparator circuit can generate only a single output pulse, and thus only the character with the highest response is indicated. A 32-wire printing device could therefore be driven directly from the comparator. However, in the present model this output is applied to a 32 \times 5\text{-}diode matrix, giving a 5-wire output which again could be used direct. For use with a standard teleprinter, the 5-wire output is applied in parallel to a 6-bit shift register, i.e. a start pulse and 5 character pulses, of conventional design.

The shifting pulses which are applied to the shift register are derived from the timing system and coincides with the transitions of a standard teleprinter waveform. Thus the final waveform which is derived from the serial output of the shift register is free from distortion.

### 2.4 Synchronization

It is implicit in the description given that the time of transition between characters is known, and this implies a system of synchronization between the input signal and the timing ‘clock’ of the receiver equipment. To obtain this, synchronizing information is added to the signal in the form of a 10c/s square-wave modulation on the a.f. tone. The depth of the modulation is small and has little effect on the reception process. In the receiver another 10c/s square wave is derived from the clock circuits, and this is designed to be exactly 90° out of phase with the modulation when the equipment is fully synchronized.

The input signal is half-wave detected and then synchronously detected by the clock square wave. Any phase error between the two generates a d.c. signal which is applied to a reactance modulator in the circuit of the master oscillator from which all the clock waveforms are derived. The circuit thus constitutes a phase-locked loop—an established technique for this purpose. The result of this process is that any phase error to zero.

### 2.5 Automatic gain control

The quality of signal on which the receiving device will function efficiently is so low that an a.g.c. voltage derived by simple detection of the input signal would be useless, as its level could be determined almost entirely by the noise or interference level.

However, the sawtooth signal which occurs in the output of the ‘wanted’ character, as shown in Fig. 3, has an effective noise bandwidth of about 10c/s only. A diode system which continuously follows the highest output of the 32 detectors will produce a voltage that, with a noise-free signal, is a recurring sawtooth of amplitude proportional to the signal, while any noise fluctuations will only represent the noise power in about 10c/s bandwidth. The signal/noise ratio at this point is therefore some 15dB better than that of the input signal, and the smoothed d.c. component may be used as a gain-controlling voltage. This voltage is also presented on the front panel meter, which therefore indicates the true working level of the resonators. In practice, this a.g.c. waveform is obtained directly from the voltage comparator circuit.

The d.c. component of this waveform is added to a 10kc/s sawtooth voltage, and the result is applied to a trigger circuit. The output of this circuit is an on-off waveform at 10kc/s, the mark/space ratio of which is a function of the controlling voltage. This waveform is used to key the a.f. signal and thus to vary the gain. To enable the fastest control to be obtained compatible with the smoothing of the a.g.c. voltage, it is important that the gain of the a.g.c. loop be kept as low as possible, and the shape of the sawtooth waveform is therefore deliberately designed to maintain the a.g.c. loop gain constant at all input levels, thus giving the lowest loop gain for a given range of control.

In this way, an a.g.c. time-constant of about 200–300ms is obtained, while the intrinsic range of about 30dB of the receiving system itself is extended to about 50dB. (In later models, the range has been further extended to more than 60dB).
in the receive equipment is rotating in exact synchronism with
the corresponding shutter in the sending equipment. Because
of the transit time of the signal between the two equipments,
there will be a phase displacement between the two shutters.
As the link will function without appreciable degradation with
a relative phase error of 20 ms, simplex working on an over-
over basis between two Piccolos is practicable on almost all
circuits. Alternatively, when a simplex Piccolo link uses
another system as the return link, e.g. Morse or speech,
reversal of the direction of Piccolo transmission is possible
without time having to be allowed for resynchronizing.

2.7 Standby facilities

In addition to the 32 character tones, the system uses
a 33rd tone. When this standby tone is sent or received, it
delivers a continuous mark to the teleprinter. This tone is
transmitted (a) when the tape-reading bulb is lifted, (b) when
there is no tape in the head, and (c) when the unit is switched
to 'receive'. This last facility means that, when both ends of
a link are working simplex, the send channels are kept open
and the receivers maintained in synchronism even when both
ends are in the receive condition.

The standby frequency is generated by the tape-head
shutter, so that it is always exactly the 65th harmonic of the
character speed, whatever that may be. When the system is
used over a carrier telephone or s.s.b. link, beat-oscillator
errors may displace the audio-tone frequencies but will not
affect the character rate. Thus, if any beat oscillator in the
system is adjusted so that the standby tone is again the
65th harmonic of the character rate, all frequency errors in
the signalling route will be compensated for.

3 Calculated performance

3.1 Error probability

3.1.1 2-tone system

Consider, in the first instance, a system such as that
described but comprising two channels only. Let a signal be
applied at the frequency of one channel, with added random
noise of uniform spectral density. It is established in
Appendix 9.2 that the probability distribution of samples
of the detected output of the signal channel (taken at times
\( t_0 \) as described above) will be the same as that of the envelope
of a continuous sine wave with added noise. Therefore, as
far as the voltage comparator circuit is concerned, the
detector output may be assumed to consist of the envelope
of a hypothetical waveform

\[ V_a = V_0 \sin \omega t + V_N \quad \ldots \quad \ldots \quad \ldots \quad (1) \]

where \( V_N \) is a random noise voltage in a narrow band
around the frequency \( \omega \) and has an r.m.s. value \( P \).

The output at time \( t_0 \) of any channel differing by a frequency
\( n f_0 \) from the signal will be zero in the absence of noise, and
as far as the voltage comparator circuit is concerned, the
unwanted channel will therefore contain noise only.
The unwanted detector output may then be considered as
the envelope of \( V_a = V'_N \), where \( V'_N \) is a random noise voltage
in a narrow band centred around the unwanted channel
frequency.

As is established in Appendix 9.2, the effective noise band-
width of either channel is \( 1/f_0 \), and therefore the r.m.s. value
of \( V'_N \) is the same as that of \( V_N \) but the two waveforms are
completely uncorrelated.

The two waveforms \( V_a \) and \( V_b \) are both similar in form to
a continuous sine wave (at their respective channel fre-
quencies) which is 'modulated' by a random voltage of mean
frequency approximately equal to \( 1/2 f_0 \).

The function of the comparator circuit is such that the
probability of identifying the signal channel incorrectly is the
probability that the envelope of \( V_a \), at the instant of comparison,
less than the envelope of \( V'_N \), and this probability is
derived in Appendix 9.3 and is shown in Fig. 4.

Fig. 4
Error probability per choice between two channels

3.1.2 2-tone 5-bit system

In the system just considered, choice is made between
two channels, and such a choice can convey only one binary
digit (or 'bit') of information. To convey one character of an
alphabet of 32, five such choices are necessary. An error in
any of these choices will result in the wrong character being
printed. The chance of this happening may be derived from
the above results by using the theorem:

If the probability of an event occurring in a single trial is \( p \),
the chance of it happening \( n \) times in \( m \) trials is

\[ P_n(n) = C^m_n p^n (1 - p)^m - n \]

Thus the total probability of getting one or more errors in a
sequence of five choices is

\[ P_e = \sum_{n=1}^{5} C_5^n p^n (1 - p)^{5-n} \quad \ldots \quad \ldots \quad \ldots \quad (2) \]

or

\[ P_e = 5p(1 - p)\left(1 + 2q + 2q^2 + q^3 + \frac{q^4}{5}\right) \]

where \( q = p(1 - p) \).

If \( p = 0.05, q = 0.05/0.95 \approx 0.05 \) and we can neglect
powers of \( q \) higher than 1. Then, since \((1 - p)^n(1 + 2q) \approx 0.905\),
these terms may also be neglected, with a consequent
inaccuracy of less than 10\%.

\[ P_e \approx 5p \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3) \]

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Therefore when \( p < 0.05 \), the character error rate for a 2-tone 5-bit system may be obtained directly from Fig. 4 by multiplying the bit error rate by five.

### 3.1.3 32-tone system

The Piccolo system employs 32 channels, and therefore five bits of information are conveyed in a single choice. As the noise in the 31 unwanted channels is uncorrelated, the error rate in comparing one wanted and 31 unwanted channels is the same as would be obtained by comparing one wanted and one unwanted channel 31 successive times.

By the same reasoning as before, it may be shown that

\[
P_e \approx 31p . . . . . . . . . . . \quad (4)
\]

The inaccuracy in this case is less than 10% if \( p < 0.003 \).

#### 3.1.4 Comparison of 2-tone and 32-tone systems

Let the information rate for both systems be 1 bit/sec, and the ratio of the input signal power to the noise spectral density be \( W \).

For the 2-tone system, \( t_0 = 1 \text{ sec} \), and the effective noise bandwidth of one channel is 1 c/s. The peak signal/r.m.s. noise in a single channel is

\[
a = \sqrt{(2W)}
\]

The corresponding character error rate is

\[
P_e = 5p
\]

Using these two conversion factors, the curve of Fig. 4 is modified to a plot of \( P_e \) against \( W \) (see Fig. 5).

For the 32-tone system, \( t_0 = 5 \text{ sec} \), and the effective noise bandwidth of a single channel is 1/5 c/s. Thus \( a = \sqrt{(10W)} \) and \( P_e = 31p \).

The corresponding error curve of \( P_e \) against \( W \) is also plotted in Fig. 5.

These curves may be used for signalling rates other than 1 bit/sec by interpreting the factor \( W \) as

\[
W = \frac{\text{signal energy per bit}}{\text{noise power per c/s bandwidth}}
\]

The actual bandwidth occupied by the two systems would be approximately \( (33/5)C = 6.6C \) for Piccolo and approximately \( 2C \) for the 2-tone 5-bit system, where \( C \) is the signalling rate in bits/sec.

### 3.2 Limitations on the application of the theory to the practical case

In practice, the ideal system on which the calculations are based cannot be realized. There are a number of practical considerations which cause departures from the theoretical performance.

(a) **Quenching time.**—The time required to quench the tuned circuits is wasted, and must be deducted from the 'sampling time'. The loss is normally about 5%. Thus, from eqn. 19, the signal/noise power ratio in the wanted channel is reduced by 5%, the noise in the unwanted channel remaining the same as that in the wanted channel. Also, the comparison of channels takes place before the adjacent channel signal voltages have reached zero.

(b) **Modulation error.**—For synchronizing purposes, in the practical system the tone is modulated with a 50/50 square wave at the character rate. The effect of this is to modify the responses of the adjacent channels so as to leave a residual voltage at the time of comparison. The total of this and the quench-time error is theoretically approximately 0.07 for 10% modulation depth; i.e. a voltage of about 7% of the wanted signal remains on the adjacent unwanted channels.

(c) **Detector errors.**—The fact that the detector time-constant is finite means that the assumption that the envelopes of the waveforms are compared is an approximation. There are three effects to be considered:

(i) the presence of 'ripple' on the wanted signal
(ii) the presence of ripple on the noise signals in other channels
(iii) the 'hold-up' of the detector reservoir capacitor preventing the output from following faithfully any rapid decreases in signal level. (This particularly applies near the end of the sampling time in the adjacent channels, as can be seen from Fig. 3.)

The practical detector time-constant is a compromise between factors (i) and (iii), and is such as to give a ripple of not more than 12% peak-to-peak on the wanted channel on a noise-free signal and reasonably close following on the adjacent channels. However, it can be seen that, when there is an appreciable probability of error, the wanted voltage and the unwanted voltage are approximately the same, so that ripple on them is of the same order of amplitude and therefore does not materially affect the error probability.

(d) **Synchronizing errors.**—Errors in the synchronizing system may cause errors in timing of the quenching and sampling functions, in that the sample of signal taken may contain, say, 90% of the wanted tone and 10% of that preceding or following it. In the practical system, with a noise input such as to give worse than 1% errors, the synchronizing 'jitter' was measured as being about 0.05 μs.
This again reduces the output of the unwanted channel by a corresponding amount and generates a spurious residual voltage of about the same amount on the channel corresponding to the preceding or subsequent character.

Other possible errors in the system include non-linearity in amplifiers causing cross-modulation and change of resonator frequency with amplitude, but in general these are negligible.

It can be seen that the total effects of all errors may be considered under three headings:

(i) Decrease in effective sampling time, \( t_0 \), due to a quench time error of about 5% and 5% synchronizing errors, say 5% mean—an effective reduction of 10% total. Thus, from eqn. 19, the effective signal/noise ratio of the wanted channel is deteriorated by about 0.4 dB.

(ii) Increase in residual voltage on adjacent channels due to quench time, modulation, detector hang-on and synchronizing errors. The sum total effect of all factors except the last has been measured at about 15% of peak wanted signal, and the additional contribution by the synchronizing error will be small.

(iii) Increase in residual voltage on one unwanted channel due to synchronizing errors will normally be less than 5% and will be negligible.

The effect of (i) may be allowed for by offsetting the curve of Fig. 5 by 0.4 dB. The effect of (ii) is more complex and could theoretically be allowed for by plotting the probability function of the unwanted channel and performing graphical or numerical integration. The important thing, however, is that these effects only increase the chances of error in the two adjacent channels, the effects on the other channels being negligible, and the results may be assessed practically in the following way. In a typical test with a noisy signal, a total of 122 errors were recorded. In an ideal system, the chances of printing an error on an adjacent channel are the same as for any other channel. Therefore the number of adjacent-channel errors should be approximately \((2/31) \times 122\), or 8. In practice, 17 such errors were counted, implying—within the accuracy of such an experiment—that the total effect of the discrepancies listed above is to double the chances of error on the adjacent channels and so increase the total chances of error by 2/31 or 6%.

Thus the predicted effects of divergences in the system consist of a decrease in effective output signal/noise ratio of about 0.4 dB and an increase in the error rate of about 6%. This latter adjustment is small, and it seems that the performance of the practical system should not diverge from its theoretical limit by more than, say, 0.6 dB.

Practical measurements gave points within \( \pm 0.5 \text{ dB} \) of the computed curve. The fact that the majority of measurements indicate a performance better than the ideal suggests a small systematic error. However, in view of the large number of factors involved, the overall agreement of about 1 dB between practical results and 'corrected' theory would seem to be reasonably satisfactory.

4 Measured performance

4.1 Random noise and fading

The performance of an early prototype was measured against added random noise for input signal levels of a few dB input signal. The measurements of the true r.m.s. voltages were made after the a.f. band-pass filter in the input to the receive system, in an effective noise bandwidth (measured) of 470 c/s, and all figures quoted are in this bandwidth. To convert to the signalling bandwidth of 340 c/s, the noise levels should be reduced by 1.4 dB.

The experiments were divided into two series. The first, the results of which are shown by the dotted lines in Fig. 6, was taken with the a.g.c. system disabled, and therefore represents conditions in which fading is so rapid that the automatic gain control cannot follow it. A particular case of this occurs on short-distance h.f. links, where long-delay multipath effects can cause sudden selective fading even in a narrow a.f. band. Two points of interest may be noted: firstly that it is possible to operate 4 dB below noise level with only 0.2% errors, and secondly that a selective fade which causes a single character tone to be received 20 dB below the normal level of 0 dBm will produce an error in only 1% of cases, provided that the noise level is not more than +3 dB on the 'dropped' level.

The second series of tests, with the a.g.c. circuit operative, is shown by the solid lines in Fig. 6, and it can be seen that

\[
\text{Fig. 6} \\
\text{Character error rate against signal level for various signal/noise ratios} \\
\begin{array}{c}
\text{with a.g.c.} \\
\text{without a.g.c.} \\
0 \text{ dB = overload point}
\end{array}
\]

the range of input is extended to about -40 dB for 'slow fades. This limit could be extended to -60 dB if necessary.

It is interesting to note that, although the standard input of 0 dBm represents the overload point of the receiving resonators, the equipment can be operated without error at +10 or +20 dBm with a signal of adequate signal/noise ratio. Very-high-level signals cannot be tolerated indefinitely, however, as overloading removes the 10 c/s modulation and causes loss of synchronizing information.

 delays (p.t.d.) of up to 4 ms at 40 fades/min caused less than 0.5 dB deterioration at low signal/noise ratios and approximately \(3 \times 10^{-1}\) residual character errors at high signal/noise ratios, while a p.t.d. of 10 ms at 20 fades/min caused less than 3 dB deterioration with a residual character error rate of approximately \(6 \times 10^{-1}\).

The authors wish to express their thanks to the Post Office Research Department for carrying out these tests and for agreeing to the publication of this note.

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4.2 Performance—impulse noise and interference

The type of interference having the worst effect consists of a steady tone, identical in frequency with one of the character frequencies. Under these circumstances, if the interfering frequency is 2 dB or more below the level of the wanted signal and white noise is low, the equipment will print substantially error-free copy. If the interfering signal is amplitude- or frequency-modulated so that tones are not sustained for periods of 100 ms or more, considerably higher levels of interference may be tolerated.

The long signalling element of 100 ms and the true integration of the signalling input power over the whole of this time can be expected to give an extremely high protection against impulse noise. No laboratory measurements have been made to date.

5 Comparison with the performance of other systems

5.1 Theoretical considerations

There are two basic methods of comparing different systems in noisy circuits.\(^1\) The first is the absolute comparison in terms of the ratio

\[
W = \frac{\text{signal energy per bit}}{\text{noise power per unit bandwidth}}
\]

for a given error rate. Thus, if the measured signal/noise power ratio is \(SN\), \(C = \text{signalling speed in bits/sec}\) and \(B = \text{bandwidth in which noise is measured}\),

\[
W = \frac{S}{N/C} = \frac{SB}{NC}
\]

Thus, if the signal/noise ratio required for a given error rate is measured for a number of different systems, the results may be reduced to a common basis by multiplying each by the appropriate factor \(B/C\).

For example, considering a system signalling at 60 words/min, giving a certain error rate at +20 dB signal/noise ratio, measured in a bandwidth of 200 c/s,

\[
C = \frac{60 \times 6 \times 5}{60} = 30\ \text{bits/sec}
\]

\[
B = 200\ \text{c/s}
\]

\[
W = \frac{S}{B/C} = \frac{200}{200}\ = 6.6\ or\ 8.3\ dB
\]

Thus, when \(S/N = 20\ dB\), \(W = 28.3\ dB\).

It is important to note that \(B\) is defined as the bandwidth in which the noise is measured, and no reference is made to the bandwidth actually occupied by the signal. Thus the factor \(W\) alone (for a specified error rate) indicates the ability of a system to signal at a certain sending rate through noise of given spectral density without reference to the signalling bandwidth.

The required bandwidth is frequently of importance, and it is necessary to consider the ‘normalized bandwidth’ \(B_0/C\), where \(B_0\) is the minimum bandwidth required to transmit the signal and does not include guard bands, allowance for oscillator drift, etc. \(B_0\) may be considered as the minimum bandwidth which will contain sufficient of the components of the theoretical signal spectrum to give negligible deterioration of performance over that obtained in an infinitely wide bandwidth.

For some systems, \(B_0\) is well defined and \(B_0/C\) is an invariant quantity for different signalling rates. Both these statements apply to the Piccolo systems (if consideration is limited to the a.f. bandwidth). For other systems, e.g. f.s.k., \(B_0\) is less well defined, and \(B_0/C\) will vary considerably with such parameters as deviation.

The maximum theoretical value of \(C/B_0\) is given by Shannon’s theorem as \(\log_2 (1 + S/N)\), where \(N\) is assumed to be in the bandwidth \(B_0\). We therefore obtain a minimum value for \(W\), namely

\[
W_0 = \frac{S}{N/C} = \frac{B_0}{B_0/C} = (2C/B_0 - 1)
\]

\(W_0\) is plotted as a function of \(B_0/C\) in Fig. 7 and is the curve designated Shannon limit. It will be seen that systems requiring small bandwidths have greater values of \(W_0\). The required bandwidth cannot be reduced indefinitely, and the limit of \(B_0/C = 0.5\) is shown in Fig. 7 as the Nyquist limit.

5.2 Comparison with the performance of various systems

During the past few years, a variety of different signalling systems have been described in the literature, and theoretical and practical performance figures have been quoted. The performance of a number of these has been reduced, as far as practicable, to a common basis, and is plotted in Fig. 7. This diagram shows \(W\) as a function of \(B_0/C\) for various error rates. The figures plotted for Piccolo were measured at audio frequency and would also apply to the performance over a suitable radio link, e.g. s.s.b. The performance at low signal/noise ratio over a radio link such as conventional d.s.b. could be degraded by 6 dB or more owing to the action of the detector.

A distinction must be drawn between a ‘signalling system’, e.g. f.s.k.—t.t.y., predicted wave, etc., which accepts information in a standard code—usually some form of binary code—and delivers it in the same code, and a ‘coding system’, e.g. Morse, bi-orthogonal, Hamming, etc., which is a method of coding information to be delivered to a signalling system.

The performance of a binary signalling system is usually given in the form of the ‘bit error rate’, in which case it has been assumed that the system is used to send a 5-bit character, i.e. an alphabet of 32 characters, and no synchronizing information, and therefore the character error rate is five times the bit error rate within the range plotted. It should be noted that, in the case where start–stop information is sent in 7-unit code, the total number of character errors is normally about 17 times the bit errors owing to multiple errors when the printer loses synchronism.\(^16\)

For coding systems the bandwidth is indeterminate as it will depend on the signalling method used. In this case, it has been assumed that the bandwidth is twice the Nyquist limit, i.e. \(B_0/C = 1\), as this is characteristic of a coherent detection system such as biphase pulse-code modulation.

Both these assumptions give decidedly optimistic results, and the generally expected trend of a practical system is indicated with an arrow on the line of plotted results.

Symbols on each curve indicate the character error rate at that point. Theoretically, each line should be curved slightly upwards at high error rates, as this implies a loss of information and therefore an increase in the factor \(B_0/C\), but as the effect is only slight, and will be the same on all curves [except possibly systems involving some forms of error detection and correction (e.d.c.)], this has been ignored.

The curves fall into two distinct categories. The theoretical curves are drawn in a single line and identified by a capital letter, each curve being mathematically derived from equations which express the ideal performance of the system discussed. The practical curves represent the measured performance of a practical system and are in double lines and

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identified with a lower-case letter. Where the practical system may be identified directly with one of the theoretical systems, the same letter is given to both curves to facilitate comparison. The practical curves are, for the most part, derived from published material, with the necessary conversion or extrapolation; the assumptions on which the conversions are based are discussed in each case. It should be emphasized that, as some systems can take various forms, the curves are not exhaustive; for example, in 2-tone systems, multiplexing is possible by interleaving channels, and this reduces the bandwidth.

**System A—Ideal systems.**—The performance of ideal systems would be specified by points on the curve marked 'Shannon limit'.

**System B—Piccolo** (32 tone).—The theoretical performance of system B has been derived in Section 3, and it has been shown that the practical system should be no more than 0.7 dB worse than the theoretical performance. Practical measurements on the system indicate a performance agreeing with the theoretical limit to within 0.5 dB; this is considered to be reasonable experimental error, and the two curves are indicated here as being coincident.

The normalized bandwidth factor $B_0/C$ is fundamental, as $B_0$ may be taken as $33 \times \text{(characters/sec)}$ and $C$ as $5 \times \text{(characters/sec)}$; therefore

$$B_0/C = 33/5 = 6.6 \text{ cycles/bit}$$

Incidentally, the spectrum power outside the limits of $B_0$ falls off extremely rapidly, allowing close spacing of channels.

**System C—Bi-orthogonal code.**—The use of orthogonal and bi-orthogonal codes is discussed by Harmuth, and his theoretical error figures are reproduced here, with the assumption of an 'optimum binary system' bandwidth giving $B_0/C = 1$. The code waveforms are relatively complex; the degree of synchronization required is high (better than 2% of the character time), and interpretation of the signal involves cross-correlation and integration of 32 complex waveforms. No practical system using this form of coding is known to the authors.

**System D—Binary phase modulation.**—It has been established that a binary system using $\pm 90^\circ$ phase modulation and perfectly synchronous coherent detection is capable of a performance that is theoretically the optimum for any system that interprets the information bit by bit, and that for such a system the normalized bandwidth $B_0/C = 1$. This is a very good standard for comparison of binary systems. The error rate given in the curve is five times the bit error rate.

**System E—Law's 'ideal telegraph receiver'.**—Law establishes that the minimum error liability theoretically attainable in a 2-tone frequency-modulation system is given by

$$P_e = \frac{1}{2} + \frac{1}{2} \text{erf} \left(\frac{W}{N}\right)^{1/2}$$

where $W$ is the energy in one signal element and $N$ is the noise power per unit bandwidth. This equation, normalized as described previously (assuming character errors = 5 x bit errors), is the basis of the figures reproduced here.

*With reference to eqn. 6, Mr. H. B. Law has pointed out that the definition of the error function originally used to derive this equation does not conform to present-day accepted practice. Using the more normal definition, i.e.

$$\text{erf} x = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) \, dt$$

the equation should read

$$P_e = \frac{1}{2} - \frac{1}{2} \text{erf} \left(\frac{W}{2N}\right)^{1/2}$$

This does not modify the curves in any way.

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The factor $B_0/C$ is not fundamental to the system as it depends on the frequency shift employed, and in Fig. 7 it has been assumed to be arbitrarily smaller than in the practical system. Law recommends this curve as a standard of comparison.

**System e—Law double-diversity.**—This curve represents the published performance\(^5\) of a double-diversity 2-tone telegraph system developed by Law et al. The system uses diversity radio reception with 2-tone f.s.k. signalling. In the receiving equipment, the clipping level is varied according to the prevalent signal/noise ratio in that particular channel, assessed over a number of previous elements. The system can theoretically derive complete information from either tone in the absence of the other.

Normalized bandwidth of this system for the parameters given by Law ($500$ c/s shift at $100$ bauds) would be a minimum of $6-8$ for a hypothetical 5-unit synchronous system and would be about $10$ for a $74$-unit start-stop system. Practical installations often use frequency shifts chosen to reduce multipath effects, usually $170$, $340$ or $510$ c/s, in which case the bandwidth is modified accordingly.

In comparing this curve with B, it should be remembered that the use of double-diversity reception with Piccolo will give a theoretical improvement of about $1\cdot4$ dB.

**System F—2-tone Piccolo.**—In the theoretical analysis of the 32-tone Piccolo system, the initial calculations give the performance of a 2-tone bit-by-bit system based on the same principles of critical spacing, total integration and comparative assessment. Comparison of this curve with B thus demonstrates the $6$ or $7$ dB of improvement derived from a multitone system. The normalized bandwidth of such a system is again a constant, independent of signalling rate, of about $2$ cycles/bit.

There are circumstances in which bandwidth conservancy is more important than performance against noise, and the 2-tone system would be suitable for this application, giving results directly comparable with present-day high-efficiency systems. In fact, both the Kineplex, h, and predicted-wave, j, systems embody some of the basic principles involved.

**System G—Hamming code.**—The application of redundancy codes is becoming increasingly important. A Hamming error-correcting code\(^6\) that adds four parity-check elements and one synchronizing element to each 8-bit of characters applied to the hypothetical system, F, may be taken as typical. The calculated results shown at G are optimistic at high error rates, as the 'error-correction' system may, under these circumstances, actually insert additional errors which are not allowed for on the curve.

It can be seen that the net result of application of the code is a negligible change in signalling efficiency. This does not imply that such systems are useless, but only that a reduction in error rate does not in itself imply an increase in communication efficiency.

**System h—Kineplex.**—A practical system described by Doelz, Heald and Martin,\(^7\) the Kineplex system, uses phase modulation of a carrier to convey binary information. The carrier energy is integrated over an element in a manner fundamentally similar to Piccolo, and the phase information is stored during the next element, during which time integration is taking place in a second identical resonator. At the end of the second element the two phases are compared, and any change of phase is interpreted as mark or space. The actual equipment carries two channels per carrier and multiplexes 20 carriers, the carrier spacing being orthogonal, as in Piccolo. The whole system is synchronous, synchronizing being by a separate carrier channel.

The practical curve is reproduced from the reference, assuming character error rate = $5 \times$ bit error rate. No theoretical analysis is given.

The normalized bandwidth of this system, which is quoted as sending $3000$ bits/sec in a bandwidth of $2$ kc/s, is only $0-66$ cycle/bit. This is based on the assumption that the spectrum of a single phase-modulated channel is small compared with the spectrum of the 20 channels, and the figures would therefore approach $B_0/C = 1$ if a system using fewer channels were used.

**System j—Predicted wave signalling.**—A second system described by Doelz\(^1\) that has much in common with the 2-tone Piccolo system, F, is predicted wave signalling. Principal differences are that the tone spacing is not orthogonal and that synchronizing information is sent as start-stop elements on a third frequency.

The system uses diversity reception with two resonators on the mark frequency and two on the space, the highest output of the four being chosen. This theoretically gives a $1\cdot4$ dB improvement in the performance. The redundancy due to transmitting synchronizing information causes about $1\cdot5$ dB deterioration, so the final results should theoretically be directly comparable with system F. Part of the deficiency of the practical results of the system may be accounted for by multiple errors due to loss of printer synchronism. The normalized bandwidth for the system quoted is about $23$ cycles/bit.

It may be noted in passing that a $7$-frequency multitone system described by Jordan\(^10\) has overall performance characteristics somewhat similar to the predicted/wave system.

**System k—Start-stop narrow-deviation f.s.k.**—Curve k, and also curve l, is taken from a paper by Watt et al.,\(^18\) that compares the measured performance of various systems under noisy conditions. This particular system had the following characteristics:

- **signalling speed**: $60$ words/min (45 bauds)
- **frequency shift**: $\pm 50$ c/s
- **receiver f.f. bandwidth**: $170$ c/s
- **receiver post-discriminator bandwidth**: $70$ c/s

The normalized bandwidth is about $3$ cycles/bit.

**System l—Start-stop wide-deviation f.s.k.**—As for k, system l has the characteristics:

- **signalling speed**: $60$ words/min (45 bauds)
- **frequency shift**: $\pm 425$ c/s
- **receiver f.f. bandwidth**: $1\cdot6$ kc/s
- **receiver post-discriminator bandwidth**: $75$ c/s

The normalized bandwidth is about $36$ cycles/bit.

This is typical of many f.s.k. teleprinter links, and it is notable that figures quoted by Law\(^14\) as common for such a link are about $2$ dB worse.

**System m—Morse.**—Curve m can be only an approximate assessment, but it is based partly on figures given by Watt et al.,\(^10\) for a 'good operator' at $15$ words/min and partly on measurements made by one of the authors on experienced operators at $10$ words/min. These latter tests were done at bandwidths of $2$ kc/s and $100$ c/s. No significant difference was observed, and the results agreed with those in the reference to within $2$ dB. It can be assumed, therefore, that the curve represents, to a fair accuracy, the performance of a good Morse operator working under adverse conditions of white noise. The normalized bandwidth will depend on the...
modulation system, but for on-off keying it is about 8 cycles/bit.

It can be seen that the Piccolo system has a theoretical and practical superiority of 6-8 dB over the more sophisticated modern systems discussed, and some 15 dB superiority over wide-deviation f.s.k. The bandwidth occupied when used with an s.s.b. link is about the same as that of a narrow-shift f.s.k. system and about 20% that of a wide-deviation f.s.k. system working at the same information rate.

When compared to a Morse operator, it has been established experimentally that the Piccolo system can send at 100 words/min with small error rate when the signal/noise ratio is so low that a Morse operator can hardly detect the presence of the signal and finds it impossible to read.

5.3 Results of field trials

A programme of field trials of the preproduction model of Piccolo is in progress. Preliminary tests have, however, been carried out using an experimental prototype equipment working at approximately 41 bauds. This equipment was similar in performance to the present prototypes, but had a more restricted a.g.c. range, longer a.g.c. time-constant and a less effective synchronizing system.

(a) Piccolo v. Hellschreiber.—The tests consisted of a comparison between Piccolo and an established long-wave (60-62 kc/s) Hellschreiber link between England and Europe that was known to give poor performance at times. The principal interference consisted of very-high-amplitude bursts of man-made noise, and there was also some deep fading. The Hellschreiber transmission was A1 (i.c.w.), and, in the absence of suitable s.s.b. equipment, the Piccolo signal was transmitted A3 (d.s.b., a.m.) with about 80% depth of modulation. Different transmitters were used, resulting in the Piccolo carrier being received at a level 10 dB less than that of the Hellschreiber. In spite of this, the Hellschreiber consistently gave two or three times the errors of the Piccolo system, except when the static was so intense as to swamp both systems.

(b) F.S.K. v. Piccolo d.s.b. (selective fading).—These tests were conducted to compare the two systems under conditions giving long-delay echoes and consequent heavy selective fading, and the radio frequency (2.75 Mc/s) and range (about 85 miles) were selected to encourage these effects. Under such conditions, echoes with delays greater than 25 ms have been measured and 5-10 ms can be quite common.

The f.s.k. (F.I) system, operating at 45 bauds (6 characters/sec) and 770 c/s total shift, modulated a 200 W transmitter. Every ten minutes the transmitter was switched to the Piccolo system, which amplitude-modulated to about 80%. The receiver aerial was switched at corresponding times between two separate receiver systems.

Results showed that, when conditions were difficult, the f.s.k. system usually produced between 3 and 10 times more character errors than did Piccolo. In one particular case where the signal dropped nearly 30 dB for about half an hour, Piccolo produced about 4% errors while the f.s.k. copy was completely unreadable.

The only circumstances under which f.s.k. showed any appreciable superiority was when selective fading effects reduced the depth of modulation of the a.m. signal virtually to zero for several seconds at a time, the f.s.k. signal remaining almost unaffected. This effect would be less troublesome with the extended a.g.c. range of the improved Piccolo equipment and with s.s.b. reception.

(c) F.S.K. v. Piccolo s.s.b. transmission (2.75 Mc/s).—The radio link in these tests was similar to that in (b), but the received signals were routed by land lines back to the transmitting terminal.

The f.s.k. was ±250 c/s shift on 2.75 Mc/s at 6 characters/sec (45 bauds). Piccolo was transmitted as s.s.b. (suppressed carrier), on 2.75 Mc/s + 126-350 c/s, at 3-4 characters/sec using a high-stability keyer. The receiver for the Piccolo signal was a specially modified s.s.b. receiver.

It was also arranged that the transmitter power (kept the same for both signals) could be reduced until appreciable errors were being printed, and most tests were carried out with a radiated power of about 50 mW.

The results of the first of these tests are as follows:

<table>
<thead>
<tr>
<th>Error rates</th>
<th>F.S.K. %</th>
<th>Piccolo %</th>
</tr>
</thead>
<tbody>
<tr>
<td>'Flat' fading, no interference</td>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td>Bad thunderstorm static and r.t.</td>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>speech interference</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bursts of r.t. speech and steady carrier interference</td>
<td>30</td>
<td>11</td>
</tr>
</tbody>
</table>

(d) F.S.K. v. Piccolo s.s.b. transmission (5.89 Mc/s).—The general circumstances were as described for (c) except for the change of radio frequency. The 3 : 1 to 10 : 1 superiority in error rate of Piccolo over f.s.k. for equal transmitter power was again reproduced, with a general tendency towards the higher ratios when the noise level was low and the fading deep.

In a second set of tests the transmitter power at each test was adjusted so as to maintain the error rate at a predetermined level, irrespective of the system in use. In general, under conditions of heavy fading, the transmitter power when using Piccolo could be reduced by 11-14 dB below that required for f.s.k.

(e) Middle East—U.K.—A short exploratory series of tests have been carried out between stations in the Middle East and England, comparing Piccolo with an established f.s.k. link.

General data are as follows:

(i) F.S.K. system:
- Transmitter power: 500 W
- Frequency shift: ±250 c/s
- Code: 74-unit start-stop
- Speed: 6 characters/sec (45 bauds)

(ii) Piccolo system:
- Transmitter power: 500 W (peak)
- Modulation: 100% amplitude modulation with square wave at tone frequency
- Speed: 5-4 characters/sec

The two transmitters were of similar design and connected to similar rhombic aerials, transmission taking place simultaneously on two radio frequencies as close together as practicable; e.g. most of the tests were carried out on 18-392 Mc/s and 18-347 Mc/s. Reception was by similar aerials and radio receivers at the same site in England.

In general, the signal was too good to allow convenient comparison, the f.s.k. recording 132 errors in 61000 characters and the Piccolo 10 errors in about the same number. There were, however, two short-duration periods of radio black-out when signals fell to a very low level. On one of these occasions the Piccolo recorded about 2% errors, and the f.s.k. about 20-30%. On the second occasion the signal was virtually inaudible, and the f.s.k. copy was estimated to contain about 3% of correct characters, the corresponding Piccolo copy being about 30% correct.
Narrowband frequency modulation.—Transmitters for f.s.k. circuits commonly do not have facilities for amplitude modulation. However, the frequency modulator is usually linear or can easily be made so, and many frequency shift receivers have linear demodulators. To determine whether Piccolo can be transmitted over circuits so equipped, tests over the circuit described in (d) have been made using frequency modulation with various deviations up to \pm 600 c/s.

The tests are still in progress, and it was found that narrower deviations gave the best results when selective frequency modulation with various deviations up to \pm 600 c/s.

6 Conclusions

The performance of the signalling system described has been shown to be superior to that of other systems at present in use on noisy radio circuits. This superiority is the result of integrating the total energy of each character over the whole character duration, spacing the tones orthogonally and avoiding the use of reference levels. The performance on very noisy circuits is better than can be achieved by a Morse operator, and allows teleprinters to be used on circuits which are at present operated manually.

In its existing form the equipment is intended for telegraph purposes using 75-baud teleprinters, but the basic principles involved have applications on multiplex teleprinter circuits and telemetry. In addition to h.f. radio circuits, the method can be applied to any other circuit where noise is a limitation, e.g. tropospheric-scatter and satellite-communication circuits.

7 Acknowledgment

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8 References


9 Appendixes

9.1 Transient response of resonant circuits

9.1.1 General case

Consider a parallel resonant circuit, \( L, C \) and \( R \), excited by a sinusoidal current. The voltage across the capacitor is given by

\[
C \frac{dv}{dr} + \frac{1}{L} \int v dt = I \sin \omega t
\]

The complete solution of this equation is

\[
v = A \exp( -at) \sin(\omega d t + B) + b \sin(\omega t + \phi).
\]

where

\[
a = 1/2CR \quad \phi = \tan^{-1} \frac{2CR(\omega - \omega_0)}{a}
\]

\[
b = \frac{R}{[(1 + \omega^2 C^2 R^2)(1 - \omega^2 C^2 R^2)]^{1/2}}
\]

\( A \) and \( B \) are constants depending on the initial conditions.

The damped frequency is given by

\[
\omega_d = \sqrt{(1/LC - 1/4C R^2)^{1/2}}
\]

The natural frequency is given by \( \omega_0 = 1/(LC)^{1/2} \).

It may be seen that the solution contains two components: (a) the 'steady state' solution, a continuous sine wave of the same frequency as the driving waveform and of amplitude proportional to \( b \); (b) the 'transient component', a sinusoidal waveform of frequency \( \omega_d \) decaying exponentially with the time-constant \( 1/a \).

Thus the envelope of the response will contain a beat frequency decaying in amplitude with a time-constant \( 1/a \), owing to the addition of the two components. The general
shape of the response, for the case when $v = \dot{v} = 0$ at $t = 0$, is shown in Fig. 8.

### 9.1.2 Lossless circuits

In the case of a lossless circuit, $R = \infty$, $a = 0$, $\omega_d = \omega_0$, and $\phi = \pi/2$. Eqn. 8 reduces to

$$v = A \sin (\omega_0 t + B) + \frac{I_0}{C(\omega_0 - \omega_0)} \sin (\omega t + \pi/2)$$

For initial conditions $v = \dot{v} = 0$ at $t = 0$, this reduces to

$$v = \frac{I_0}{C(\omega + \omega_0)(\omega - \omega_0)} (\cos \omega_0 t - \cos \omega t)$$

If $\Delta \omega = \omega - \omega_0$ and $\Delta \omega \ll \omega$,

$$v = \frac{I_0}{C} \frac{2}{\Delta \omega t} \sin \Delta \omega t \sin \left(\frac{\omega + \omega_0}{2} t\right)$$

In the limiting case, when $\Delta \omega = 0$,

$$v = \frac{I_0}{2C} \sin \omega t$$

i.e. the envelope of the output rises linearly with time.

In the circuit of Fig. 2a, all transformers are of the same turns ratio and are driven by the same current. Therefore, from eqn. 11, if the tuning capacitance is made the same for all frequencies, the sensitivity of each circuit, expressed as the ratio of the detector output to the drive current at the resonant frequency, will be the same.

The envelope of eqn. 10 is plotted in Fig. 9 for various frequencies, and in Fig. 10 the value of this envelope at a fixed time, $t_0$, is plotted as a function of frequency. The latter curve has the form $|\sin x/x|$ and represents the overall frequency response of the channel, provided that consideration is restricted to observations of the output at $t = t_0$.

Two things will be noticed: first that this response is different from the steady-state response of the circuit, which for zero losses is infinitely narrow, and secondly that the response is zero for all frequencies differing from $\omega_0$ by $m/2\omega_0$, where $m$ is an integer.

### 9.1.3 Lossy circuits—limits of linearity

For a circuit with finite but low losses, such that $\omega_d \approx \omega_0$, it can be shown that, when $\omega = \omega_0$, the solution to eqn. 2 is approximately

$$v = IR \left[1 - e^{-t/(2CR)}\right] \sin \omega t$$

Comparison of eqn. 11 and eqn. 12 shows that the effect of losses is to reduce the output from the lossless value $v_0$ by a non-linearity error:

$$\frac{v_0 - v}{v_0} = 1 - \frac{2CR}{t} \left[1 - e^{-t/(2CR)}\right]$$

that is

$$\frac{v_0 - v}{v} = \frac{(t/2CR)}{2} - \frac{(t/2CR)^2}{6} + \frac{(t/2CR)^3}{24}$$

Therefore, for $t \ll CR$, the non-linearity error is approximately $t^4/4CR^4$, and this applies whether $R$ is positive or negative, i.e. whether the circuit is damped or quasi-oscillatory. Expressed in alternative parameters, the error is approximately $\omega_0^2/4Q_0$ or $m^2/2Q_0$, where $m$ is the number of cycles of signal frequency over which integration takes place.

Although losses in the tuned circuit are effectively cancelled by the positive feedback circuit, the foregoing relationships are of importance when the effects of changes in coil losses or feedback circuit gain are considered. For instance, let the natural $Q$-factor of the tuned circuit be $Q_0$ and the effective parallel resistance be $R_p$. Assume that the feedback circuit has been adjusted to cancel these losses, and let the $Q$-factor of the coil deteriorate to $Q_p(1 - \alpha)$ owing to some factor such as temperature, humidity, etc. The loss resistance of the coil will then be effectively shunted by a further resistance, $R_p/\alpha$, and the effective $Q$-factor of the complete circuit will be reduced from infinity to $Q_p/\alpha$. The resulting non-linearity error will then be

$$\frac{v_0 - v}{v} = \frac{\alpha m}{2Q_0} = \frac{\alpha t}{4CR}$$

Consider a practical case, in which a low-frequency coil of copper wire (whose d.c. resistance is assumed to constitute the majority of the circuit losses) must operate over a temperature range of 50°C with a non-linearity error not worse than 5%.

The resistance variation will be $50 \times 0.004$, i.e. 20%, and therefore, if the circuit is adjusted for zero losses at the median temperature, $\alpha = 0.1$.
Thus, if \( m = 66 \), we find that \( 0.05 = 0.1766/2<2 \) and \( \Phi_0 = 200 \). That is, either the natural (2-factor of the circuit must be higher than 200, or some form of temperature compensation must be employed in the feedback circuit.

The effects of changes in the gain of the feedback circuit may be similarly assessed.

9.2 Probability distribution of resonator outputs in the presence of noise

In Reference 6 it is shown that, if the power spectrum of the response of a bandpass filter is \( P(f) \) and has a value \( P_0 \) at the centre frequency \( f_0 \), the effective noise bandwidth \( B \) is given by

\[
B = \frac{1}{P_0} \int_{-\infty}^{\infty} P(f) df
\]

From Fig. 10 and eqn. 10, it can be seen that the spectrum of the Piccolo system is symmetrical about \( f_c \), where \( A/ = f - f_c \); therefore

\[
B = 2 \frac{1}{P_0} \int_{0}^{\infty} P(\Delta f) d(\Delta f) \quad \ldots \ldots \quad (15)
\]

From eqn. 10, the power spectrum of the envelope of the responses at a fixed time \( t_0 \) is given by

\[
p(\Delta f) = \frac{1}{2} \left( \frac{I_0}{2C} \right)^2 \quad \ldots \ldots \quad (16)
\]

where \( x = \frac{2\pi\Delta f}{t} \)

Thus

\[
P_0 = \frac{1}{2} \left( \frac{I_0}{2C} \right)^2 \quad \ldots \ldots \quad (17)
\]

Therefore

\[
B = 2 \int_{0}^{\infty} \left( \frac{\sin x}{x} \right)^2 (d\Delta f)
\]

and, as \( x = \pi\Delta f_0, dx = \pi\Delta f_0(d\Delta f) \)

\[
B = \frac{2}{\pi t_0} \quad \ldots \ldots \quad (18)
\]

i.e. the effective noise bandwidth is the reciprocal of the sampling time.

In practice, owing to the effects of noise or errors in synchronization, the drive current \( i \), and therefore \( v \), may not be zero when \( t = 0 \), but if \( t_0 \gg 2\pi/\omega_0 \), the resulting error in \( B \) is negligible.

The r.m.s. response of the circuit to random noise is the same as that which would be obtained by passing the noise through a perfect filter of bandwidth \( B \). Therefore, if the input current \( i \) has an r.m.s. value \( I \), measured in a bandwidth \( B_n \), the r.m.s. voltages, obtained by sampling at times \( t_0 \), are given by

\[
\frac{I}{2C} B_n = \frac{1}{2C} \frac{B}{B_n} = \frac{I}{2C} \frac{t_0}{B_n} \quad \ldots \ldots \quad (19)
\]

If the input current consists of mixed signals and noise, \( i = I_0 \sin \omega t + I \) [having a signal/noise power ratio of \( AI/2I^2 \)], the samples produced would have the same probability distribution as that of a sine-wave-plus-noise voltage \( I_0/2C \sin \omega t + (I/2C)\sqrt{t_0/B_n} \) which has a signal/noise power ratio

\[
\frac{A^2}{2I^2} B_n t_0 \quad \ldots \ldots \quad (19)
\]

It is evident that corresponding samples of the envelope of the output would have the same probability distribution as those of the envelope of the sine-wave-plus-noise.

9.3 Error probability

9.3.1 2-tone system

The probability density \( p(v) dv \) of a voltage \( V \) is defined as the probability that a random sample of \( V \) will be between \( v \) and \( (v + dv) \).

When the waveform consists of mixed sine wave and random noise in an ideal bandpass filter

\[
V = V_0 \sin \omega t + V_N
\]

where the r.m.s. value of \( V_N \) is \( \Phi \).

The probability density of the envelope is

\[
p_\Phi(v) dv = v \exp \left( -\frac{v^2}{2} \right) \frac{I_0}{\Delta f_0} \sin x dv \quad \ldots \ldots \quad (20)
\]

where \( v = V_0/\Phi, a = V_0/\Phi \), and \( I_0 \) is the Bessel function with imaginary argument.

The corresponding function for the envelope of the narrow-band noise alone is

\[
p_\Phi(v) dv = v \exp \left( -\frac{v^2}{2} \right) \quad \ldots \ldots \quad (20)
\]

From these functions in principle it is possible to obtain a 'distribution function' \( P(v) = \int_{0}^{\infty} p_\Phi(v) dv \), which gives the probability that \( V \) is less than \( v \). For the envelope of random noise in a narrow band

\[
P_\Phi(v) = 1 - \exp \left( -\frac{v^2}{2} \right) \quad \ldots \ldots \quad (22)
\]

There is no corresponding simple analytical expression for \( P_\Phi(v) \), but Rice gives curves for this function (derived by numerical integration) for various values of \( a \).

It is easy to show that the probability that \( V_a < V_b \) is given by

\[
P(ab) = \int_{0}^{\infty} P_\Phi(v) p_\Phi(v) dv \quad \ldots \ldots \quad (22)
\]

or

\[
P(ab) = 1 - \int_{0}^{\infty} P_\Phi(v) p_\Phi(v) dv \quad \ldots \ldots \quad (22)
\]

Fig. 4, showing the variation of \( P(ab) \) with the signal/noise ratio, was obtained by graphical integration of this expression. This curve therefore gives the probability of error at each selection in a comparison between two channels.

9.3.2 Double-diversity 2-tone system

Consider a system in which comparison is made between four channels, two of which are signal channels and two unwanted. Assume that

(a) the signal level in the two wanted channels would be the same in the absence of noise

(b) the noise level in all channels is uncorrelated.

For an error to be printed, the level in both wanted channels must be below that in either of

(c) It has been pointed out that an analytical solution to eqn. 22 has been derived and that

\[
P(ab) = \frac{1}{4} \exp \left( -\frac{a^2}{4} \right)
\]

Divergence between this curve and that shown in Fig. 4 is negligibly small.

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the unwanted channels. It can be proved that the probability of error under these circumstances is given by

\[
P(ab) = \int_0^\infty [(Pa(v))^2 pb(v)[2 - p_b(v)] dv.
\] (24)

This function, obtained graphically, is plotted in Fig. 4, and it can be seen that the effect of double diversity is to increase the effective signal/noise ratio by approximately 16%, i.e. 1.3 dB.

Provided that the requirements of (a) and (b) are fulfilled, this calculation applies equally whether both wanted channels are at the same audio frequency, e.g. space diversity, or at different frequencies—in which case the total transmitter power required is doubled.

9.4 Specification of current Piccolo model

9.4.1 Mechanical and environment specifications

Overall dimensions, in cabinet: 22 X 21 X 26 in (56 X 53 X 66 cm)

Weight, less cabinet: 110 lb approximately (50 kg)

Weight, in cabinet: 170 lb approximately (77 kg)

Maximum ambient temperature: 50°C

9.4.2 Electrical

The equipment uses transistors throughout.

Power supply: 190–250 V, 50 c/s

Tolerance on supply voltage: +10 to –20% of nominal

Power consumption: approximately 60 W

Information input: standard 5-hole fully perforated 11/16 in (1.17 cm) paper tape

Signal characteristics: continuous a.f. tone of varying frequency in the band 330–650 c/s; level nominally constant, but modulated about 10% at 10 c/s, 50/50 square wave

Output level, send: greater than 1 mW in 600 Ω

Input level, receive: 1 mW from 600 Ω source into 600 Ω internal load. The equipment will function on signals fluctuating between approximately –50 dBm and +20 dBm

Telegraph output: fully regenerated, 40 mA at 22 V to centre-tapped teleprinter magnet

Alternative outputs: 32 wire or 5 wire; 50 μs negative going pulses; 4 V peak in less than 200 Ω

9.4.3 Signalling route requirements

Frequency response.—For optimum operation under noisy conditions, the response of all equipment in the signalling channel should be level ±0.5 dB between 320 and 660 c/s. With a signal/noise ratio better than 0 dB, operation will be satisfactory with a frequency response level ±3 dB.

A.F. tolerance.—The difference in frequency between a tone generated by the send equipment and the corresponding tone delivered to the receiving equipment should not be greater than ±1 c/s if full signal/noise performance is to be obtained. This may be extended to ±4 c/s with corresponding deterioration in performance.

If circuit alignment using the standby tone can be employed, the initial error may be ±4 c/s.