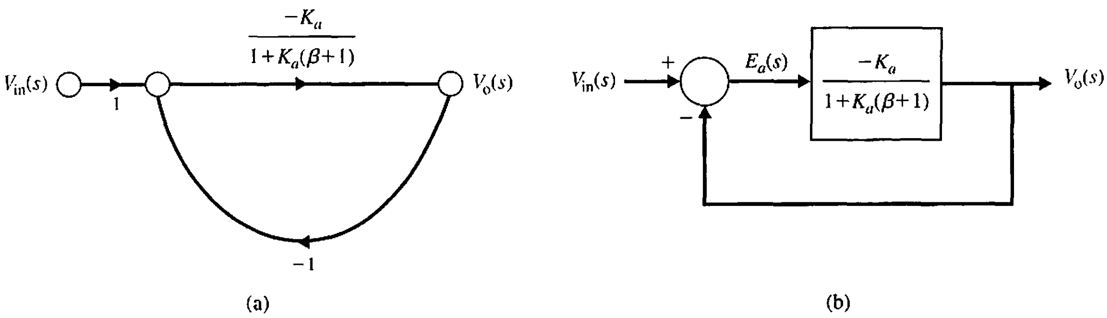

FIGURE 4.5 Block diagram model of feedback amplifier assuming $R_p \gg R_0$ of the amplifier.

and

$$R_p = R_1 + R_2. \tag{4.21}$$

The closed-loop transfer function of the feedback amplifier is

$$T = \frac{-K_a}{1 + K_a\beta}. \tag{4.22}$$

The sensitivity of the closed-loop feedback amplifier is

$$S_{K_a}^T = S_G^T S_{K_a}^G = \frac{1}{1 + K_a\beta}. \tag{4.23}$$

If K_a is large, the sensitivity is low. For example, if

$$K_a = 10^4$$

and

$$\beta = 0.1, \tag{4.24}$$

we have

$$S_{K_a}^T = \frac{1}{1 + 10^3}, \tag{4.25}$$

or the magnitude is one-thousandth of the magnitude of the open-loop amplifier.

We shall return to the concept of sensitivity in subsequent chapters. These chapters will emphasize the importance of sensitivity in the design and analysis of control systems. ■

4.4 DISTURBANCE SIGNALS IN A FEEDBACK CONTROL SYSTEM

An important effect of feedback in a control system is the control and partial elimination of the effect of disturbance signals. A **disturbance signal** is an unwanted input signal that affects the output signal. Many control systems are subject to extraneous disturbance signals that cause the system to provide an inaccurate output. Electronic amplifiers have inherent noise generated within the integrated circuits or transistors;

radar antennas are subjected to wind gusts; and many systems generate unwanted distortion signals due to nonlinear elements. The benefit of feedback systems is that the effect of distortion, noise, and unwanted disturbances can be effectively reduced.

Disturbance Rejection

When $R(s) = N(s) = 0$, it follows from Equation (4.4) that

$$E(s) = -S(s)G(s)T_d(s) = -\frac{G(s)}{1 + L(s)}T_d(s).$$

For a fixed $G(s)$ and a given $T_d(s)$, as the loop gain $L(s)$ increases, the effect of $T_d(s)$ on the tracking error decreases. In other words, the sensitivity function $S(s)$ is small when the loop gain is large. We say that large loop gain leads to good disturbance rejection. More precisely, for good disturbance rejection, we require a large loop gain over the frequencies of interest associated with the expected disturbance signals.

In practice, the disturbance signals are often low frequency. When that is the case, we say that we want the loop gain to be large at low frequencies. This is equivalent to stating that we want to design the controller $G_c(s)$ so that the sensitivity function $S(s)$ is small at low frequencies.

As a specific example of a system with an unwanted disturbance, let us consider again the speed control system for a steel rolling mill. The rolls, which process steel, are subjected to large load changes or disturbances. As a steel bar approaches the rolls (see Figure 4.6), the rolls are empty. However, when the bar engages in the rolls, the load on the rolls increases immediately to a large value. This loading effect can be approximated by a step change of disturbance torque. Alternatively, the response can be seen from the speed–torque curves of a typical motor, as shown in Figure 4.8.

The transfer function model of an armature-controlled DC motor with a load torque disturbance was determined in Example 2.5 and is shown in Figure 4.7, where it is assumed that L_a is negligible. Let $R(s) = 0$ and examine $E(s) = -\omega(s)$, for a disturbance $T_d(s)$.

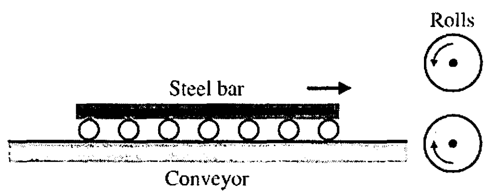

FIGURE 4.6 Steel rolling mill.

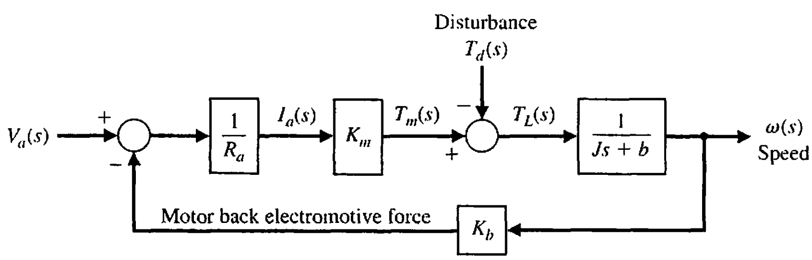

FIGURE 4.7 Open-loop speed control system (without tachometer feedback).

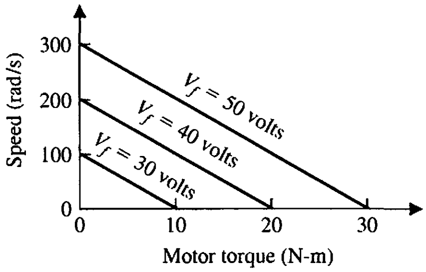

FIGURE 4.8 Motor speed–torque curves.

The change in speed due to the load disturbance is then

$$E(s) = -\omega(s) = \frac{1}{Js + b + K_m K_b/R_a} T_d(s). \tag{4.26}$$

The steady-state error in speed due to the load torque, $T_d(s) = D/s$, is found by using the final-value theorem. Therefore, for the open-loop system, we have

$$\lim_{t\to\infty} E(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \frac{1}{Js + b + K_m K_b/R_a}\left(\frac{D}{s}\right)$$
$$= \frac{D}{b + K_m K_b/R_a} = -\omega_0(\infty). \tag{4.27}$$

The closed-loop speed control system is shown in block diagram form in Figure 4.9. The closed-loop system is shown in signal-flow graph and block diagram form in Figure 4.10, where $G_1(s) = K_a K_m/R_a$, $G_2(s) = 1/(Js + b)$, and $H(s) = K_t + K_b/K_a$. The error, $E(s) = -\omega(s)$, of the closed-loop system of Figure 4.10 is:

$$E(s) = -\omega(s) = \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)} T_d(s). \tag{4.28}$$

Then, if $G_1G_2H(s)$ is much greater than 1 over the range of s, we obtain the approximate result

$$E(s) \approx \frac{1}{G_1(s)H(s)} T_d(s). \tag{4.29}$$

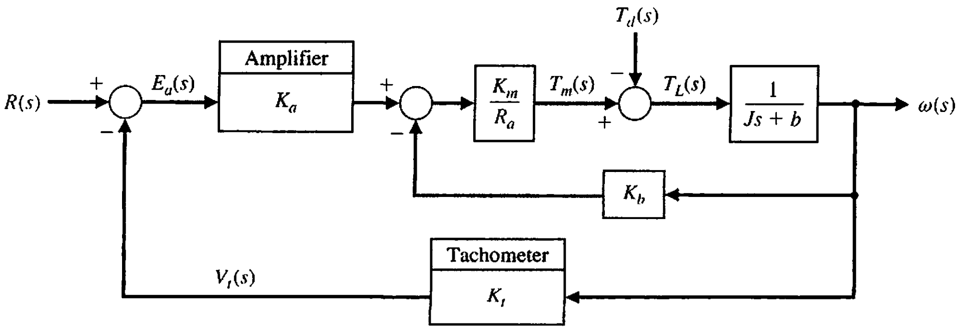

FIGURE 4.9 Closed-loop speed tachometer control system.

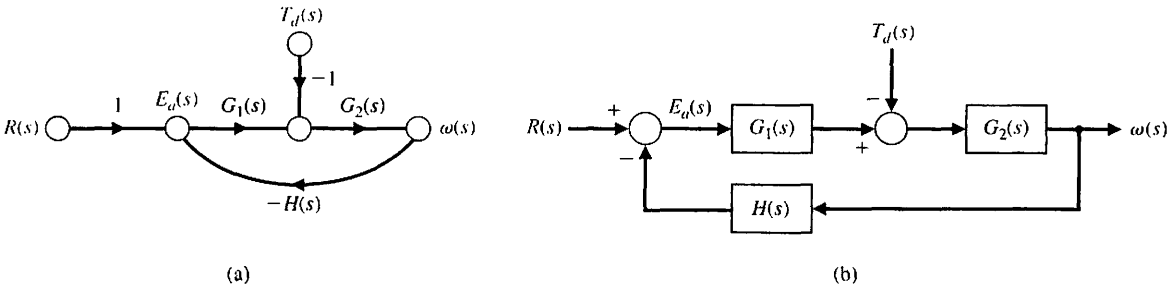

FIGURE 4.10 Closed-loop system. (a) Signal-flow graph model. (b) Block diagram model.

Therefore, if $G_1(s)H(s)$ is made sufficiently large, the effect of the disturbance can be decreased by closed-loop feedback. Note that

$$G_1(s)H(s) = \frac{K_aK_m}{R_a}\left(K_t + \frac{K_b}{K_a}\right) \approx \frac{K_aK_mK_t}{R_a},$$

since $K_a \gg K_b$. Thus, we strive to obtain a large amplifier gain, K_a, and keep $R_a < 2\ \Omega$. The error for the system shown in Figure 4.10 is

$$E(s) = R(s) - \omega(s),$$

and $R(s) = \omega_d(s)$, the desired speed. For calculation ease, we let $R(s) = 0$ and examine $\omega(s)$.

To determine the output for the speed control system of Figure 4.9, we must consider the load disturbance when the input $R(s) = 0$. This is written as

$$\begin{aligned}\omega(s) &= \frac{-1/(Js + b)}{1 + (K_tK_aK_m/R_a)[1/(Js + b)] + (K_mK_b/R_a)[1/(Js + b)]}T_d(s)\\ &= \frac{-1}{Js + b + (K_m/R_a)(K_tK_a + K_b)}T_d(s).\end{aligned} \tag{4.30}$$

The steady-state output is obtained by utilizing the final-value theorem, and we have

$$\lim_{t\to\infty}\omega(t) = \lim_{s\to 0}(s\omega(s)) = \frac{-1}{b + (K_m/R_a)(K_tK_a + K_b)}D; \tag{4.31}$$

when the amplifier gain K_a is sufficiently high, we have

$$\omega(\infty) \approx \frac{-R_a}{K_aK_mK_t}D = \omega_c(\infty). \tag{4.32}$$

The ratio of closed-loop to open-loop steady-state speed output due to an undesired disturbance is

$$\frac{\omega_c(\infty)}{\omega_0(\infty)} = \frac{R_ab + K_mK_b}{K_aK_mK_t} \tag{4.33}$$

and is usually less than 0.02.

This advantage of a feedback speed control system can also be illustrated by considering the speed–torque curves for the closed-loop system, which are shown in

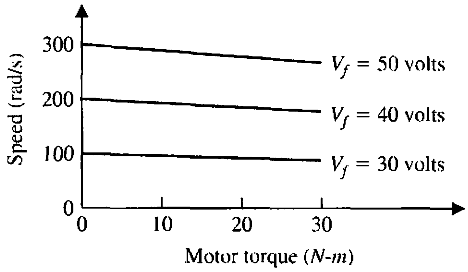

FIGURE 4.11 The speed-torque curves for the closed-loop system.

Figure 4.11. The improvement of the feedback system is evidenced by the almost horizontal curves, which indicate that the speed is almost independent of the load torque.

Measurement Noise Attenuation

When $R(s) = T_d(s) = 0$, it follows from Equation (4.4) that

$$E(s) = C(s)N(s) = \frac{L(s)}{1 + L(s)}N(s).$$

As the loop gain $L(s)$ decreases, the effect of $N(s)$ on the tracking error decreases. In other words, the complementary sensitivity function $C(s)$ is small when the loop gain $L(s)$ is small. If we design $G_c(s)$ such that $L(s) \ll 1$, then the noise is attenuated because

$$C(s) \approx L(s).$$

We say that small loop gain leads to good noise attenuation. More precisely, for effective measurement noise attenuation, we need a small loop gain over the frequencies associated with the expected noise signals.

In practice, measurement noise signals are often high frequency. Thus we want the loop gain to be low at high frequencies. This is equivalent to a small complementary sensitivity function at high frequencies. The separation of disturbances (at low frequencies) and measurement noise (at high frequencies) is very fortunate because it gives the control system designer a way to approach the design process: the controller should be high gain at low frequencies and low gain at high frequencies. Remember that by low and high we mean that the loop gain magnitude is low/high at the various high/low frequencies. It is not always the case that the disturbances are low frequency or that the measurement noise is high frequency. For example, an astronaut running on a treadmill on a space station may impart disturbances to the spacecraft at high frequencies. If the frequency separation does not exist, the design process usually becomes more involved (for example, we may have to use notch filters to reject disturbances at known high frequencies). A noise signal that is prevalent in many systems is the noise generated by the measurement sensor. This noise, $N(s)$, can be represented as shown in Figure 4.3. The effect of the noise on the output is

$$Y(s) = \frac{-G_c(s)G(s)}{1 + G_c(s)G(s)}N(s), \tag{4.34}$$

which is approximately

$$Y(s) \simeq -N(s), \tag{4.35}$$

for large loop gain $L(s) = G_c(s)G(s)$. This is consistent with the earlier discussion that smaller loop gain leads to measurement noise attentuation. Clearly, the designer must shape the loop gain appropriately.

The equivalency of sensitivity, S_G^T, and the response of the closed-loop system tracking error to a reference input can be illustrated by considering Figure 4.3. The sensitivity of the system to $G(s)$ is

$$S_G^T = \frac{1}{1 + G_c(s)G(s)} = \frac{1}{1 + L(s)}. \tag{4.36}$$

The effect of the reference on the tracking error (with $T_d(s) = 0$ and $N(s) = 0$) is

$$\frac{E(s)}{R(s)} = \frac{1}{1 + G_c(s)G(s)} = \frac{1}{1 + L(s)}. \tag{4.37}$$

In both cases, we find that the undesired effects can be alleviated by increasing the loop gain. Feedback in control systems primarily reduces the sensitivity of the system to parameter variations and the effect of disturbance inputs. Note that the measures taken to reduce the effects of parameter variations or disturbances are equivalent, and fortunately, they reduce simultaneously. As a final illustration, consider the effect of the noise on the tracking error:

$$\frac{E(s)}{T_d(s)} = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)} = \frac{L(s)}{1 + L(s)}. \tag{4.38}$$

We find that the undesired effects of measurement noise can be alleviated by decreasing the loop gain. Keeping in mind the relationship

$$S(s) + C(s) = 1,$$

the trade-off in the design process is evident.

4.5 CONTROL OF THE TRANSIENT RESPONSE

One of the most important characteristics of control systems is their transient response. The **transient response** is the response of a system as a function of time. Because the purpose of control systems is to provide a desired response, the transient response of control systems often must be adjusted until it is satisfactory. If an open-loop control system does not provide a satisfactory response, then the process, $G(s)$, must be replaced with a more suitable process. By contrast, a closed-loop system can often be adjusted to yield the desired response by adjusting the feedback loop parameters. It is often possible to alter the response of an open-loop system by inserting a suitable cascade controller, $G_c(s)$, preceding the process, $G(s)$, as shown in Figure 4.12. Then it is necessary to design the cascade transfer function, $G_c(s)G(s)$, so that the resulting transfer function provides the desired transient response.