Theory and Operating Characteristics of the Thermionic Amplifier

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Classic Paper

The theory of operation of the three-electrode thermionic vacuum tube given in this paper is based on the fundamental equation of the family of characteristic curves for various plate and grid voltages. This equation, which may be written

$$I = a\left(\frac{1}{\mu_0}\Sigma E_B + \Sigma E_c + \varepsilon\right)^2$$

where $\boldsymbol{E_b}$ *and* $\boldsymbol{E_c}$ *are the plate and grid voltages respectively, and* $\boldsymbol{a}$, $\boldsymbol{\mu_o}$ *and* ε *are the structural parameters of the device, is obtained empirically with the help of the relation previously discovered by the author, which states that the voltage between filament and plate bears a linear relation to the effective voltage produced by it between the filament and a plane coincident with that of the grid. From this it follows that an electromotive force* $\boldsymbol{e}$ *impressed upon the grid circuit produces an electromotive force* $\boldsymbol{\mu_o e}$ *in the plate circuit. With the help of these fundamental relations the amplification equations of the tube are derived in terms of the structural parameters of the tube and the constants of the circuit.*

Methods are given for experimentally determining the constants of the tube, the two most important of which are the amplification constant $\boldsymbol{\mu_o}$ *and the internal output impedance.*

Experiments are described which were performed to test these equations, and they indicate that the first order approximation made give results sufficiently accurate for amplification purposes.

I. Introduction

The three-electrode thermionic tube has been responsible for a great deal of the recent rather remarkable developments in the art of radio communication. In its most commonly known form it consists of an evacuated vessel containing a hot filament cathode, an anode placed at a convenient distance from the cathode and a third electrode in the form of a grid placed between cathode and anode. To discuss in detail the theory of operation of the device in its various applications, such as oscillation generator, radio detector, and amplifier would be beyond the scope of the present paper. What I intend to give here is merely its fundamental principles of operation, with particular reference to its application as an amplifier. The framework of this theory was worked out in the winter of 1913–14 and formed the basis of a considerable amount of research and development work that has since been done in this laboratory on the device and its various applications.

A condition which is assumed in the elaboration of the views expressed in the following is that the operation of the device is independent of any gas ionization, or in other words, that the current is carried almost entirely by the electrons emitted from the hot cathode. It is, of course, to be understood that it is at present impossible completely to eliminate ionization by collision of the electrons emitted from the cathode with the residual gas molecules. But the condition assumed can always be realized practically by evacuating the tube to such an extent that the number of positive ions formed by collision ionization is always small compared with the number of electrons moving from cathode to anode. This happens when the mean free path of the electrons in the residual gas becomes large compared with the dimensions of the device. The pressure necessary for this is not very low, and were it not for the gases occluded in the electrodes and walls of the vessel, it would be a comparatively simple matter to make the tube operate independently of gas ionization. The energy liberated by the electrons striking the anode, however, usually causes a sufficient rise in the temperature of the device to liberate enough gas to increase the pressure unduly.[1]

This is especially marked in the case of tubes handling large amounts of power. It is, therefore, necessary to denude the electrodes and walls of the tube of gases during the process of evacuation. Furthermore, since the energy liberated at the anode increases with the applied voltage, it is seen that this voltage must be kept within limits depending upon the degree of evacuation obtained. This is very important when using the device as a telephone relay, as was recognized by Dr. Arnold of this laboratory in the early stages of his experiments with this type of device. As is well known to workers in this field, it is difficult to keep

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[1] For a fuller explanation of the effect of gas, see H. J. van der Bijl, "Phys. Rev.," (2), 12, page 174, 1918.

a discharge steady and reproducible when ionization by collision is appreciable, and steadiness and reproductibility are conditions which must be complied with by a telephone relay.

The success of the tubes developed by the Western Electric Company is mainly due to the extensive study that has been made of the bearing of the structural parameters of the device on its operation. It is hardly possible to meet the requirements of efficiency and satisfactory operation of the device without an explicit mathematical formulation of its operation. A satisfactory telephone relay must, for example, do more than merely utilize the direct current power in its local circuit to amplify alternating current power: it must faithfully reproduce the incoming speech currents, it must also be capable of handling sufficient power, and have a definite impedance that can conveniently be made to fit the impedance of the telephone line. Since all these conditions depend on the structural parameters of the amplifier, they will not be satisfied unless the amplifier be properly designed, and so much distortion may be produced as to make the device worthless as a telephone relay. On the other hand, it has been found that by properly designing the amplifier the above-named requirements can be met very satisfactorily.

II. Current–Voltage Characteristics of Simple Thermionic Devices

We shall not here enter into a discussion of the extensive investigations that have been carried out on thermionics, but merely, for the purpose of elucidation, touch upon those phases of the subject which have a direct bearing on the theory of operation of the thermionic amplifier.

Consider a structure consisting of a heated cathode and an anode, and contained in a vessel which is evacuated to such an extent that the residual gas does not play any part in the current convection from cathode to anode. The number of electrons emitted from the cathode is a function of its temperature. If all the electrons emitted from the cathode pass to the anode, the relation between the resulting current I and cathode temperature T is given by a curve of the nature shown in Fig. 1. This curve is obtained provided the voltage between anode and cathode is always high enough to drag all the electrons to the anode as fast as they are emitted from the cathode; that is, I in Fig. 1 represents the saturation current. The saturation current is obtained in the following way: Suppose the cathode be maintained at a constant temperature T_1 and the voltage V between anode and cathode be varied. As this voltage V is raised from zero, the current I to the anode at first increases, the relation between V and I being represented by the curve OA_1 of Fig. 2. Any increase in V beyond the value corresponding to A_1 causes no further increase in I, and we get the part A_1B_1 of the curve. Clearly this part of the curve corresponds to the condition when all the emitted electrons are drawn to the anode as fast as they are emitted from the cathode. If the cathode temperature be increased to T_2, the number of emitted electrons is increased, and we

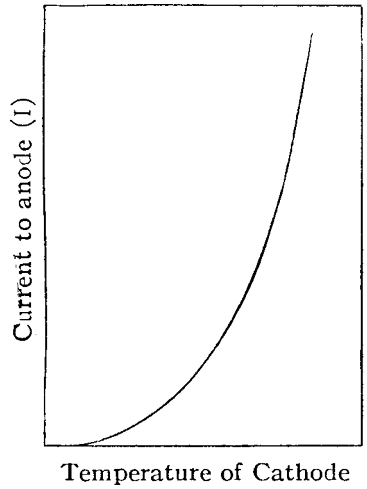

Fig. 1.

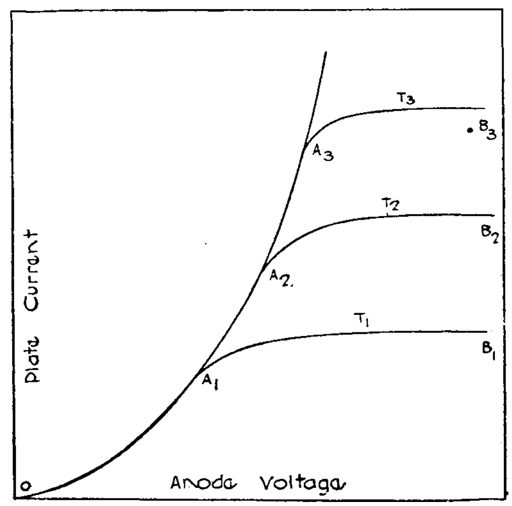

Fig. 2.

get the curve OA_2B_2. When these values of the saturation current are plotted as a function of the temperature, we obtain the curve of Fig. 1. This curve is represented very approximately by the equation

$$I = aT^{\frac{1}{2}}e^{\frac{b}{T}} \tag{1}$$

where a and b are constants. This equation was derived by O. W. Richardson in 1901[2] on the basis of the theory that the electrons are emitted from the hot cathode without the help of any gas, but solely in virtue of their kinetic energy. The formulation of this theory was the first definite expression of what may be termed a pure electron emission.

In the state in which Richardson's equation holds the current is independent of the voltage. Under these conditions the device to be treated in the following does not function as amplifier or detector, since it depends for its operation on the variation of current produced by variation of the voltage. In this device the current established in the circuit connecting filament and anode (that is, the so-called

[2] "Proc. Camb. Phil. Soc.," volume II, 285, 1901; "Phil. Trans. Roy. Soc.," A, 201, 1903.

output circuit) by the electrons flowing from filament to anode is varied by potential variations applied between the filament and grid. The condition under which the current is a function of the voltage is represented by the part OA of Fig. 2. Here the voltage is not high enough to draw all the electrons to the anode as fast as they are emitted from the cathode; in other words, there are more electrons in the neighborhood of the cathode than can be drawn away by the applied voltage. It was first pointed out explicitly by C. D. Child in 1911 that this limitation to the current is due to the space charge effect of the electrons in the space between anode and cathode. The influence of space charge is something which must always be considered where conduction takes place by means of dislodged electrons or ions, such as the conduction thru gases at all pressures, liquids, and high vacua. Assuming that in the space only ions of one sign are present, Child deduced the equation[3]

$$I = \frac{1}{9\pi}\sqrt{\frac{2e}{m}} \cdot \frac{V^{\frac{3}{2}}}{x^2}. \tag{2}$$

In this equation, which was deduced on the assumption that both cathode and anode are equipotential surfaces of infinite extent, I is the thermionic current per square centimeter of cathode surface, V the voltage between anode and cathode, x the distance between them, and e and m the charge and mass of the ion, respectively.

When the full space charge effect exists, the current is independent of the temperature of the cathode. This can be understood more easily with reference to Fig. 3, which gives the current as a function of the temperature of the cathode for various values of the voltage between anode and cathode. Suppose a constant voltage V_1 be applied between anode and cathode, and the temperature of the cathode be gradually increased. At first when the temperature is still low, the voltage V_1 is large enough to draw all the emitted electrons to the anode, and an increase in the temperature results in an increase in the current. This gives the part OC_1 of the curve of Fig. 3. When the temperature corresponding to C_1 is reached, so many electrons are emitted that the resulting volume density of their charge causes all other emitted electrons to be repelled, and these return to the filament. Obviously any further increase in the temperature of the cathode beyond that given by C_1 causes no further increase in the current, and we obtain the horizontal part C_1D_1. If, however, the voltage be raised to V_2, the current increases, since more electrons are now drawn away from the supply at the filament, the full space charge effect being maintained by less emitted electrons being compelled to return to the filament. It is now clear that the part OC of Fig. 3 corresponds to the part AB of Fig. 2 and CD of Fig. 3 to OA of Fig. 2. The latter represents the condition under which the thermionic amplifier operates.

[3] C. D. Child, "Phys. Rev.," 32, 498, 1911. The space effect has been fully studied by J. Lilienfeld ("Ann. d. Phys.," 32, 673, 1910); I. Langmuir ("Phys. Rev.," (2), 2, 450, 1913), who also independently derived the space charge equation (2) and published a clear explanation of the limitation of current by the space charge; and Schottky ("Jahrb. d. Rad. u. Elektronik," volume 12, 147, 1915).

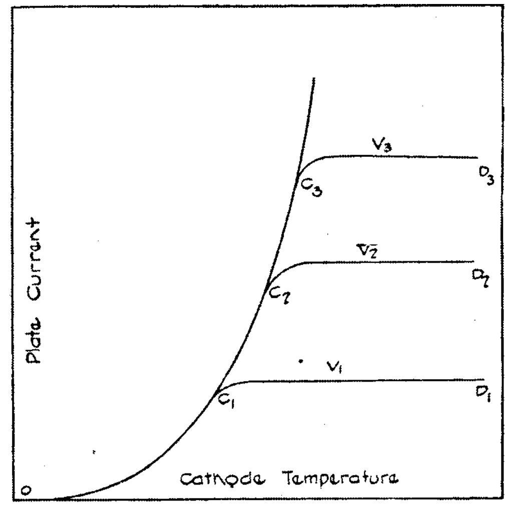

Fig. 3.

It is important to note that the thermionic amplifier operates under the condition characterized by the circumstances that the applied voltage is not sufficiently high to give the saturation current.

III. Action of the Auxiliary Electrode

So far we have considered the case of a simple thermionic device consisting of a cathode and anode. When a third electrode is added to the system, the matter becomes more complicated.

The insertion of a third electrode to control the current between cathode and anode is due to de Forest.[4] De Forest later gave this electrode the form of a grid placed between cathode and anode.[5] About the same time von Baeyer[6] used an auxiliary electrode in the form of a wire gauze to control thermionic discharge. The gauze was placed between the thermionic cathode and the anode.

The quantitative effect of the auxiliary electrode was first given by the present writer.[7]

To get an idea of the effect of the auxiliary electrode consider the circuit shown in Fig. 4. F denotes the cathode, P the anode, and G the auxiliary electrode which is in the form of a grid between F and P. Let the potential of F be zero, and that of P be maintained positive by the battery E, and let E_c for the present be zero. Now, although there is no potential difference between F and G, the electric field between F and G is not zero, but has a finite value which depends upon the potential of P. This is due to the fact that the potential of P causes a stray field to act thru the openings of the grid. If the potential of P be E_B the

[4] DeForest, U.S. Patent number 841,387, 1907.

[5] DeForest, U.S. Patent number 879,532, 1908.

[6] von Baeyer, "Verh. d. D. Phys. Ges.," 7, 109, 1908.

[7] H. J. van der Bijl, "Verh. d. D. Phys. Ges.," May, 1913, page 338. In these experiments which were also performed under such conditions that the current was carried almost entirely by electrons, the source of electrons was a zinc plate subjected to the action of ultra-violet rays. It is obvious that the action of the auxiliary electrode is independent of the nature of the electron source. Hence the results then found apply also to the present case.

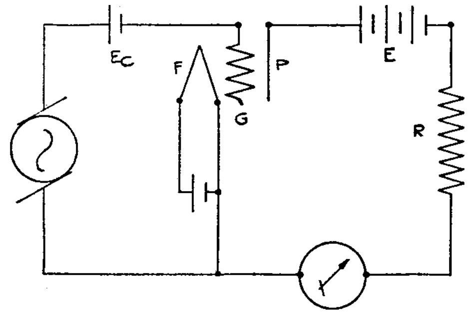

Fig. 4.

field at a point near F is equal to the field which would be sustained at that point if a potential difference equal to γE_B were applied directly between F and an imaginary plane coincident with the plane of G, where γ is a constant which depends on the mesh and position of the grid. If the grid is of very fine mesh γ is nearly zero, and if the grid be removed—that is, if we have the case of a simple valve—γ is equal to unity.

These results can be expressed by the following equation:

$$E_s = \gamma E_B + \varepsilon. \tag{3}$$

Here ϵ is a small quantity which depends upon a number of factors, such as the contact potential difference between cathode and grid and the power developed in the filament, which is the usual form of cathode used. It is generally of the order of a volt and can be neglected when it is small compared with γE_B. Obviously the current between anode and cathode depends on the value of E_s.

Now, suppose a potential E_c be applied directly to the grid G, the cathode F remaining at zero potential. The current is now a function of both E_s and E_c

$$I = \Phi(E_s, E_c). \tag{4}$$

Before determining the form of this function, let us consider in a general way how the current is affected by E_s and E_c. We have seen that E_s is due to the voltage E_B between anode and cathode, and is less than E_B if the grid is between anode and cathode, since in this case γ is always less than unity. Under the influence of E_s the electrons are drawn thru the openings of the grid and are thrown on to the anode by the strong field existing between grid and anode. The effect of E_c on the motion of the electrons between F and G is similar to that of E_s. Whether or not electrons will be drawn away from the cathode depends on the resultant value of E_s and E_c. If $E_s + E_c$ is positive, electrons will flow away from the cathode, and if $E_s + E_c$ is zero or negative, all the emitted electrons will be returned to the cathode, and the current thru the tube will be reduced to zero. Now, $E_s + E_c$ will be positive: 1) when E_c is positive (E_s is always positive), and 2) if E_c is negative and less than E_s.

i) When E_c is positive, some of the electrons moving toward the grid are drawn to the grid, while the rest are drawn thru the openings of the grid to the anode under the influence of E_s. The relative number of electrons going thru and to the grid depends upon the mesh of the grid, diameter of the grid wire, and the relative values of E_s and E_c. When, for example, E_s is large compared with E_c, the number of electrons going to the grid is comparatively small, but for any fixed value of E_s the current to the grid increases rapidly with increase in E_c. Hence, for positive values of E_c, current will be established in the circuit FGE_c, Fig. 4.

ii) If, however, E_c is negative and less than E_s, as was the case in the above named experiments of the writer, nearly all the electrons drawn away from the filament pass to the plate, practically none going to the grid. In this case the resistance of the circuit FGE_c is infinite.

If, now, an alternating emf. be impressed upon the grid so that the grid becomes alternately positive and negative with respect to the cathode, the resistance of the circuit FGE_c, which may be referred to as the input circuit, will be infinite for the negative half cycle and finite and variable for the positive half cycle. If, on the other hand, the alternating emf. be superimposed upon the negative value, E_c, the values of these voltages being so chosen that the resultant potential of the grid is always negative with respect to the cathode, the impedance of the input circuit is always infinite.

Broadly speaking, the operation of the thermionic amplifier is as follows: The current to the anode we have seen is a function of E_s and E_c, or keeping the potential E_B of the anode constant, the current for any particular structure of the device is a function only of the potential on the grid. Hence, if the oscillations to be repeated are impressed upon the input circuit, variations in potential difference are set up between cathode and grid, and these cause variations in the current in the circuit FPR, the power developed in the load R being greater than that fed into the input circuit. It is seen then that the device functions broadly as a relay in that variations in one circuit set up amplified variations in another circuit unilaterally coupled with the former.

IV. Current–Voltage Characteristic of the Thermionic Amplifier

Equation (2) which gives the current to the anode as a function of the applied voltage in the case of a simple device containing equipotential electrodes of infinite extent is of little use in deriving the amplification equations of the thermionic amplifier. In the first place, the cathode in this device is not an equipotential surface, but a filament which is heated by passing a current through it. Secondly, the insertion of a grid between the filament and the anode so complicates the electric field distribution that a theoretical deduction of the relation between the current to the anode and the applied voltages between filament and grid and filament and anode is difficult and leads to expressions that are too complicated for practical use. I have, therefore, found it more practical to determine the characteristic of the tube empirically, and found as the result of a large number of experiments that the characteristic can be represented with sufficient accuracy by the following equation:

$$I = a(E_s + E_c)^2 \tag{5}$$

where a is a constant depending on the structure of the device.[8]

With the help of equation (3) this becomes

$$I = a(\gamma E_B + E_c + \varepsilon)^2. \tag{6}$$

This gives the current to the anode as a function of the anode and grid potentials, the potential of the filament being zero. If a number of voltages be impressed upon the grid and anode, we have generally

$$I = a(\gamma \Sigma E_B + \Sigma E_c + \varepsilon)^2. \tag{7}$$

If, for example, an alternating emf., $e \sin pt$ be superimposed upon the grid-voltage, E_c, the equation becomes

$$I = a(\gamma E_B + E_c + e \sin pt + \varepsilon)^2. \tag{8}$$

It must be understood that equation (6) gives the direct characteristic of the device itself; that is, E_B in equation (6) is the voltage directly between the filament and the anode P (Fig. 4). If the resistance R be zero, E_B is always equal to E, the voltage of the battery in the circuit $EPRE$, which is constant. If R be not zero, the potential difference established between the ends of R by the current flowing in it makes E_B a function of the current. The effect of the resistance R on the characteristic will be explained later. For the present we shall confine ourselves to a discussion of the characteristic of the amplifier itself. This characteristic can always be obtained experimentally by making R equal to zero and using an ammeter in the circuit $FPER$ (Fig. 4), the resistance of which is small compared with the internal output resistance of the amplifier itself.

A graphical representation of equation (6) is given in Fig. 5. The curves give the current to the anode as a function of the grid voltage E_c for different values of the parameter, E_B.

Referring to equation (6) and Fig. 5, we see that the current is finite for negative values of the grid voltage E_c, and is only reduced to zero when

$$E_c = -(\gamma E_B + \varepsilon).$$

Differentiating I (equation 6), first with respect to E_B, keeping E_c constant, and then with respect to E_c, keeping E_B constant, we get

$$\frac{\partial I}{\partial E_B} = 2a\gamma(\gamma E_B + E_c + \varepsilon) = Q \tag{9}$$

$$\frac{\partial I}{\partial E_c} = 2a(\gamma E_B + E_c + \varepsilon) = S. \tag{10}$$

Hence

$$\frac{Q}{S} = \gamma = \text{constant} \tag{11}$$

[8] Although this equation is sufficiently accurate when using the device as an amplifier, its accuracy does not suffice for purposes of detection, since the detection action is a function of the second derivative of the characteristic.

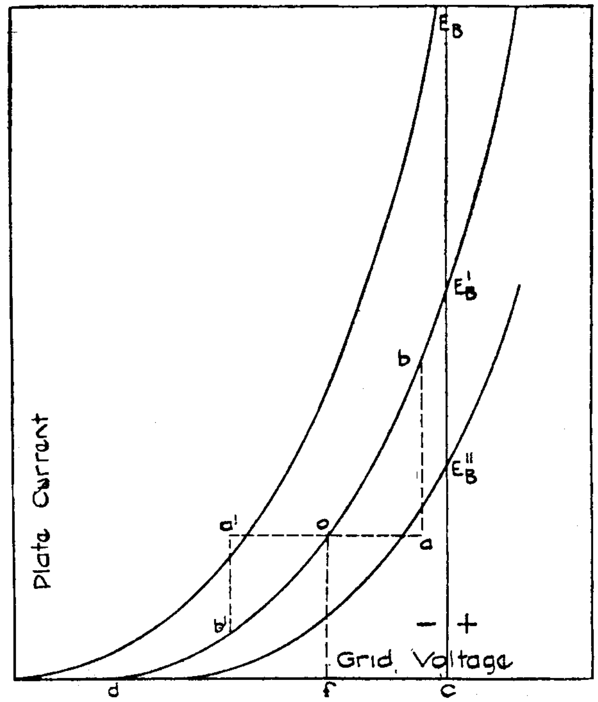

Fig. 5.

from which it follows that for equivalent values of E_B and E_c, a change in the anode voltage E_B produces γ times as great a change in the current to the anode as an equal change in the grid voltage E_c.

The output impedance of the tube is obtained from the admittance K which is given by

$$K = \frac{1}{2\pi}\int_0^{2\pi} \frac{\partial I}{\partial E_B}\, dt$$

or putting in the value of $\partial I/\partial E_B$ from (9)

$$K = \frac{1}{2\pi}\int_0^{2\pi} 2a\gamma(\gamma E_B + E_c + \varepsilon + e \sin pt)\, dt.$$

It is seen that $\partial I/\partial E_B$ is not constant but depends upon the instantaneous value of the input voltage $e \sin pt$. This is also obvious since the characteristic is curved. The admittance and impedance, however, are independent of the input voltage, as is seen readily by integrating the expression for K

$$R_0 = \frac{1}{K} = \frac{1}{2a\gamma(\gamma E_B + E_c + \varepsilon)}. \tag{12}$$

Comparing this with equation (9), it is seen that the impedance can readily be obtained by taking the slope of the characteristic at a point corresponding to the direct current values E_B and E_c at which it is desired to operate the tube.

Equation (12) can be expressed in a more convenient form by multiplying its numerator and denominator by $(\gamma E_B + E_c + \varepsilon)$

$$R_0 = \frac{\gamma E_B + E_c + \varepsilon}{2a\gamma(\gamma E_B + E_c + \varepsilon)^2}$$

which, with the help of equation (6) becomes

$$R_0 = \frac{E_B + \mu_0(E_c + \varepsilon)}{2I} \tag{13}$$

where

$$\mu_0 = \frac{I}{\gamma}. \tag{14}$$

We shall see that μ_0 is the maximum voltage amplification obtainable from the device.

Comparing (12) with (9) it is seen that $R_0 = 1/Q$ and therefore from (11) and (16) the slope of the I, E_c-curve is given by

$$S = \frac{\mu_0}{R_0}. \tag{15}$$

This constant is very important. It will be shown later that the quality of the device is determined by the value of S, that is, the slope of the curve giving the current to the plate as a function of the grid voltage.

V. Experimental Determination of the Constants of the Tube and Verification of the Characteristic Equation

In order experimentally to verify equation (6) it is necessary to know the value of the constants γ and ε. Both these constants can be determined by methods which do not depend on the exponent of the equation. The linear stray field relation

$$E_s = \gamma E_B + \varepsilon \tag{3}$$

which is involved in equation (6) is also independent of the exponent. The constants γ and ε can be determined and the relation (3) tested as follows.

Let us assume an arbitrary exponent β for equation (6)

$$I = a(\gamma E_B + E_c + \varepsilon)^\beta. \tag{16}$$

Taking the general case in which both E_B and E_c are variable, we have:

$$\frac{dI}{dE_c} = \frac{\partial I}{\partial E_B}\frac{dE_B}{dE_c} + \frac{\partial I}{\partial E_c}.$$

now

$$\frac{\partial I}{\partial E_B} = a\beta\gamma(\gamma E_B + E_c + \varepsilon)^{\beta-1}$$
$$\frac{\partial I}{\partial E_c} = a\beta(\gamma E_B + E_c + \varepsilon)^{\beta-1}.$$

Hence

$$\frac{dI}{dE_c} = a\beta(\gamma E_B + E_c + \varepsilon)^{\beta-1}\left(\gamma\frac{dE_B}{dE_c} + 1\right). \tag{17}$$

Now, let I be constant, then

$$\gamma E_B + E_c + \varepsilon = 0$$

that is

$$-E_c = \gamma E_B + \varepsilon = E_s \tag{18}$$

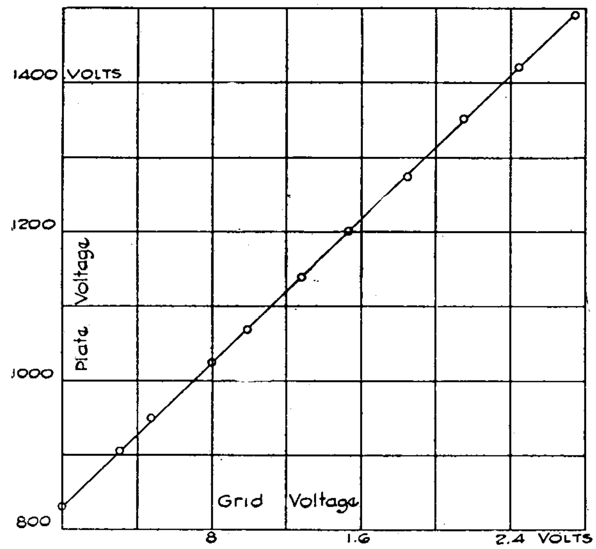

Fig. 6.

or

$$\frac{dE_B}{dE_c} = -\frac{1}{\gamma}.$$

Integrating and putting $1/\gamma$ equal to μ_0, we get

$$E'_B = E_B + \mu_0 E_c. \tag{19}$$

Equations (18) and (19) are, therefore, independent of the exponent of (6). Equation (18) gives the case in which the current has the constant value zero. It shows that E_s in equation (3) is simply the absolute value of the grid potential E_c, which suffices to reduce the current to the anode to zero when the anode has a potential E_B. (The potentials are referred to that of the filament which is supposed to be grounded.) Referring to Fig. 5, we see that equation (18) gives the relation between the intercepts of the curves on the axis of grid potential E_c and the corresponding values of anode potentials E_B. This is the method which I used several years ago to test the linear stray field relation (3). The accuracy with which this relation is obeyed is seen from Figs. 3 and 5 of my above mentioned publication.[9]

The factor μ_0, which plays a very important part in the theory of operation of the thermionic amplifier, can be obtained by taking the slope of the curve giving the relation between E_B and E_c in accordance with equation (18). It can be more easily determined with the help of equation (19), which gives the relation between the anode and grid potentials necessary to maintain the current at some convenient constant value. Fig. 6 gives results obtained in this manner. The linear relation obtained between E_B and E_c verifies equation (19).

[9] "Vehr. d. D. Phys. Ges.," above. For further experimental verification of this relation when applied to the case in which the cathode consists of a hot filament, see H. J. van der Bijl, "Phys. Rev.," (2), 12, 184, 1918.

Another method of determining μ_0 is with the help of equation (11)

$$\frac{Q}{S} = \gamma = \frac{1}{\mu_0}. \tag{11}$$

S is the slope of the curve giving the current to the anode as a function of the grid potential, and Q the slope of the curve which gives the current as a function of the anode potential. Since both these slopes depend upon the anode and grid potentials E_B and E_c, they must be measured for the same values of E_B and E_c. This method gives quite reliable results but is not as convenient as the one explained above.

The most convenient method of measuring the amplification constant μ_0 is that recently given by Miller.[10] The principle of this method is the same as one that has been frequently used in this laboratory where it is necessary to determine μ_0 for a large number of tubes. The circuit shown in Fig. 7 is contained in a box with terminals for the ammeter A and batteries E_B and E_A. The tube is plugged into a socket provided for it in the box. It is seen from the previous paragraph that a voltage in the grid circuit is equivalent to μ_0-times that voltage in the plate circuit. Hence, referring to Fig. 7, it is evident that no change will be produced in the ammeter A on closing the key K, if $r_1/r_2 = \mu_0$. For convenience in measurement r_2 is a fixed value of 10 ohms, and r_1 consists of three dial rheostats of 1 000 100, and 10 ohms arranged in steps of 100, 10, and 1 ohms each. The rheostats are marked in tenths of the actual resistances, so that the setting of the dials read the μ_0 directly. The drain of the battery E_1 is very small because the circuit is only closed momentarily by the push button K. This battery, therefore, consists of small dry cells enclosed in the box. Instead of using a direct current supplied by the battery E_1, an alternating current can be used, in which case the ammeter A must be replaced by a telephone receiver. The use of an alternating current has the advantage that it allows a simple determination of the impedance of the tube according to the method given by Miller. Fig. 8 shows a photograph of the μ_0-meter with a tube inserted in its socket. The rheostat R enables the filament current to be adjusted to the desired value.

In order to test the characteristic equation (6) it is still necessary to know the value of ε. This can be obtained by applying a convenient negative potential to the grid and keeping it constant while observing the current to the anode for various values of the anode potential. The grid being negative with respect to the filament, no current could be established in the filament-grid circuit. There should be no resistance in the circuit FPE except that of the ammeter, and this should be small compared with the internal output resistance of the amplifier itself. Under these conditions the voltage of the battery E is always equal to E_B, the voltage between filament and anode, so that the observed values of current and voltage give the true characteristic of the amplifier. From the curve giving the current as a function

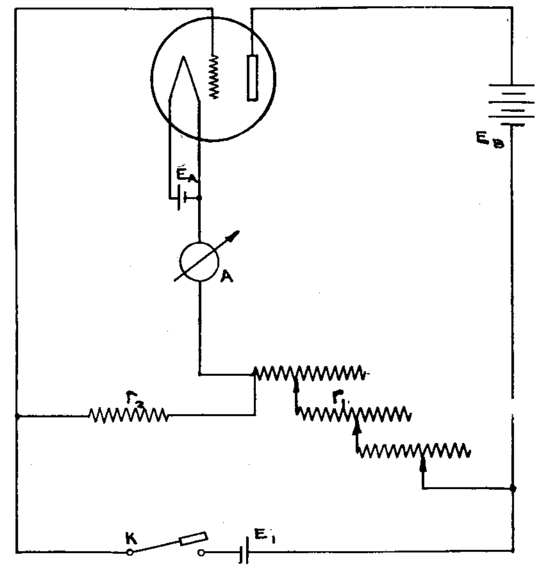

Fig. 7.

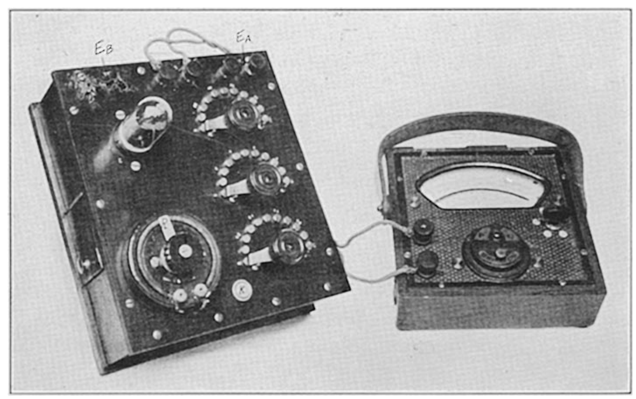

Fig. 8.

of the anode potential the value of the anode potential can be determined for which the current is reduced to zero. This potential, of course, depends upon that of the grid. By putting I equal to zero in equation (6), and $\gamma = 1/\mu_0$ we get

$$\varepsilon = -\left(\frac{E_B}{\mu_0} + E_c\right).$$

Once μ_0 and ε are known, the current can be plotted against the expression

$$\left(\frac{E_B}{\mu_0} + E_c + \varepsilon\right)^2 \tag{20}$$

Fig. 9 shows the results for one particular type of tube. In this case the grid had a constant negative potential equal to ε. Hence the current was simply plotted as a function of E_B. The straight line gives the relation between E_2^B and the anode current. It is seen that the parabolic relation of the fundamental equation (6) is obeyed with sufficient accuracy.[11]

[10] J. M. Miller, Proceedings of the Institute of Radio Engineers, volume 6, page 141, 1918.

[11] For further results of these experiments, see "Phys. Rev.," (2), 12, 186, 1918.

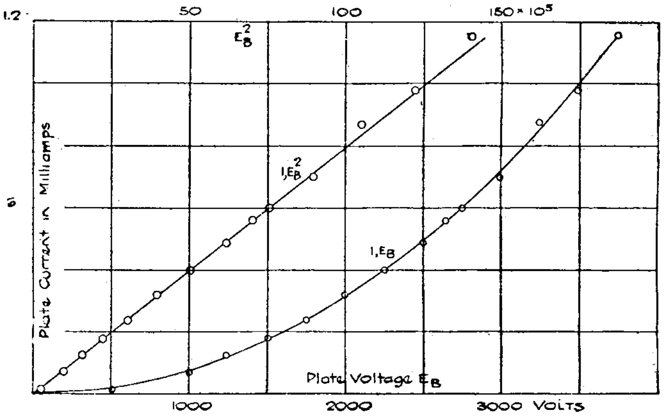

Fig. 9.

VI. Characteristic of Circuit Containing Thermionic Amplifier and Ohmic Resistance in Series

In discussing the behavior of the thermionic amplifier in an alternating current circuit, we shall make two assumptions.

First. The alternating current established in the circuit $FPER$ (Fig. 4) is a linear function of the voltage impressed upon the input circuit FGE_c. This implies that the power amplification is independent of the input. This is the condition for an ideal amplifier.

Second. The thermionic amplifier shows no reactance effect. This implies that if the amplifier be inserted in a noninductive circuit, the power amplification produced is independent of the frequency.

No proof is needed to establish the validity of the second assumption. The first is, however, not true except under certain conditions, and it remains to determine these conditions and operate the amplifier so that they are satisfied. Let the external resistance of the output circuit FPE (Fig. 4) be zero, and the resistance of the current-measuring device negligibly small compared with the internal output impedance of the amplifier itself. Under these circumstances E_B, which denotes the potential difference between filament and anode, is independent of the current in the circuit FPE, and always equal to the voltage E of the battery in the circuit. Hence, if the current be plotted as a function of E_c, the potential difference between filament and grid, the parabola given by equation (6) and Fig. 5 is obtained. If, now, the potential of the grid be varied about the value E_c equal to cf (Fig. 5), it is obvious from the curve that the increase, ab, in current to the anode due to a decrease, oa, in the negative potential of the grid is greater than the decrease $a'b'$ in current caused by an equal increase, oa', in the negative grid potential. In this case, the output current consists of the following parts: let the alternating input voltage superimposed upon E_c be $e \sin pt$ then

$$I_0 = a(\gamma E_B + E_c + \varepsilon + e \sin pt)^2. \tag{8}$$

Expanding this we get

$$I_0 = a(\gamma E_B + E_c + \varepsilon)^2 + 2a(\gamma E_B + E_c + \varepsilon)e \sin pt + \frac{ae^2}{2}\cos(2pt + \pi) + \frac{ae^2}{2}. \tag{21}$$

The first term represents the steady direct current maintained by the constant voltages E_B and E_c when the input voltage e is zero (equation 6). The second term gives the alternating output current oscillating about the value of direct current given by (6). It is in phase with and has the same frequency as the input voltage. When using the device as an amplifier, this is the only useful current we need to consider. The first harmonic represented by the third term is present, as was to be expected in virtue of the parabolic characteristic. The last term, which is proportional to the square of the input voltage, represents the change in the direct current component due to the alternating input voltage, and is the only effective current when using the device as a radio wave detector. If a direct current meter were inserted in the output circuit, it would show a current which is greater than that given by equation (6) by an amount equal to $ae^2/2$, the last term of equation (21). This is the state of matters when the device works into a negligibly small resistance.

If, on the other hand, the output circuit contains an appreciable resistance,[12] R, the voltage E_B between filament and plate is not constant, but is a function of the current, and an increase in the current due to an increase in the grid potential sets up a potential drop in the resistance R, with the result that E_B decreases, since the battery voltage E is constant. E_B is now given by

$$E_B = E - RI. \tag{22}$$

In order to obtain the characteristic of the circuit containing the tube and a resistance R, let us substitute (22) in (8)

$$I_0 = a[\gamma(E - RI) + E_c + \varepsilon + e \sin pt]^2$$

and put $\gamma E + E_c + \varepsilon = V$. This gives

$$I_0 = \frac{1 + 2a\gamma R(V + e \sin pt) - \sqrt{1 + 4a\gamma R(V + e \sin pt)}}{2a\gamma^2 R^2} \tag{23}$$

This expression can be expanded into a Fourier series

$$I_0 = \begin{cases} \left(\frac{1+2a\gamma RV - \sqrt{1+4a\gamma RV}}{2a\gamma^2 R^2}\right) \\ + \left(1 - \frac{1}{\sqrt{1+4a\gamma RV}}\right)\frac{e}{\gamma R}\sin pt \\ + \frac{(-1)^{n+1}2^n(2n-1)a^n\gamma^{n-1}R^{n-1}e^{n+1}\sin^{n+1}pt}{n+1(1+4a\gamma RV)^{2n+\frac{1}{2}}} \end{cases} \tag{24}$$

From this it is seen that the rate of convergence of the series increases as R is increased. Actual computations show that when the tube is made to work into an impedance equal to or greater than that of the tube the harmonic terms become negligibly small compared with the second term of (24), which is the only useful term when using the tube as

[12] The insertion of a suitable resistance in the output circuit to straighten out the characteristic and so reduce distortion was, I believe, first suggested by Dr. Arnold, who also showed experimentally that distortion is almost negligible when the external resistance is equal to the impedance of the tube.

an amplifier, so that we can assume that the amplification is independent of the input voltage.[13]

When the tube works into a large external resistance it can show a blocking or choking effect on the current. This is seen from the following: the voltage E_B which is effective in drawing electrons thru the grid to the anode is given by equation (22). If now the current be increased, not by increasing the electromotive force in the circuit FPR (Fig. 5), but by increasing the potential difference between filament and grid, the current I increases, while E, the electromotive force in the plate circuit, remains constant, from which it follows that E_B must decrease while E_c, the grid voltage, increases. The result of this is that more electrons that otherwise would have come thru the grid to the anode are now drawn to the grid. If the input voltage becomes large enough the anode circuit FPR may be robbed of so many of its electrons that no further increase in current in the anode circuit results no matter how much the grid voltage is increased. Under these conditions equation (24) does not apply. Its application is limited to the conditions stated by equations (25) and (26).

Even if the series represented by the last term of equation (24) were zero, distortionless transmission can only be obtained if the input voltage is kept within certain limits.

Let the input voltage, $e \sin pt$, be superimposed upon the negative grid voltage, E_c (Fig. 5). Theoretically speaking, one condition of operation is that the grid should never become so much positive with respect to the filament that it takes appreciable current, for if this happens the current established in the grid circuit would lower the input voltage, and therefore the amplification. In actual practice the extent to which the grid can become positive before taking appreciable current depends upon the value of the plate voltage and the structure of the tube. We can therefore state that a condition for distortionless transmission is $e \leqq |E_c| + |g|$, where g is the positive voltage which the grid can acquire without taking enough current to cause distortion. Another condition is that the input voltage must not exceed the value given by df (Fig. 5); otherwise the negative peaks of the output current wave will be chopped off. Now cd is given by $\gamma E_B + \varepsilon$. This is obtained by equating the current I to zero in equation (6). We therefore have the conditions

$$|e| \leqq E_c| + |g|, \qquad |e| \leqq |\gamma E_B + \varepsilon| - |E_c| \tag{25}$$

or when the tube is working at full capacity—that is when operating over the whole curve

$$e = |E_c| + |g| = |\gamma E_B + \varepsilon| - |E_c|. \tag{26}$$

It may be remarked here that when using the tube as an oscillation generator, these limits are not obeyed. From equation (12), it is seen that the impedance R_0 of the tube is independent of the input voltage $e \sin pt$. This is, however, true only as long as the characteristic is parabolic. Referring to Fig. 5, if the input voltage oscillates about the value f, the slope of the tangent at o is a measure of the impedance, in fact, it is μ_0 divided by the impedance (equation 15). Since the characteristic is parabolic, it follows that the secant thru bb' is always parallel to the tangent at o as long as $oa = oa'$ (equal to the input voltage e). The impedance can, therefore, be obtained by taking the slope of the secant thru the maximum and minimum current values. If now the tube works beyond the limits of the parabolic characteristic, such as along the curve OAB (Fig. 2) the slope of this secant does not remain constant for all values of the input voltage, so that the impedance of the tube is not independent of the strength of the oscillations but increases with it, the minimum impedance being obtained when the oscillations are infinitely small. This is what happens when the tube operates as an oscillation generator. Part of the energy in the output circuit is fed back to the input circuit, thus increasing the strength of the oscillations in the output until the tube works beyond the limits of the parabolic characteristic. The impedance of the tube is thereby increased, and this increase continues until the impedance acquires the maximum value capable of sustaining the oscillations, consistent with the degree of coupling used between the output and input circuits. It is readily seen that the current obtained in the output is not a single pure sine wave, but contains a number of harmonics as well. The extent to which these harmonics influence the current values obtained, when the latter is measured simply by the insertion of a hot wire meter in the output circuit, which, of course, measures the total current, depends on the degree of coupling as well as the constants of the output circuit. These considerations must be borne in mind when dealing with the alternating current output power obtainable from an oscillation tube. The only useful power is, of course, that which is due to the fundamental.

VII. Amplification Equations of the Thermionic Amplifier

On the strength of the two assumptions discussed in the previous paragraph, namely, that the amplification is independent of the input and the frequency, it is possible to derive the equations of amplification in a very simple way. Referring to Fig. 4, let the current in the external resistance R be varied by variations produced in the grid potential, E_c. Then, as was shown in the last paragraph, E_B is also a variable depending on the current I, as shown by

$$E_B = E - RI \tag{22}$$

[13] In this connection I want to point out that although the parabolic relation used here represents the characteristic of the tube with sufficient accuracy when using the tube as an amplifier, and indeed with quite a good degree of accuracy, as shown by the experimental curves, yet the approximation is not close enough to represent accurately the second and higher derivatives of the characteristic, and therefore, too much reliance should not be placed on the actual values of the several harmonic terms represented by the last term of equation (24). This equation is merely intended to show, as it does, in a general way how the insertion of a resistance in the output circuit of the tube tends to straighten out the characteristic.

where E is the constant voltage of the battery in the output circuit $FPER$. Hence

$$I = \Phi(E_B, E_c)$$

from which

$$\frac{dI}{dE_c} = \frac{\partial I}{\partial E_B} \cdot \frac{dE_B}{dE_c} + \frac{\partial 1}{\partial E_c}.$$

This gives the variation of current in R as a function of the variation in the grid voltage.

Substituting from (9) and (10)

$$\frac{dI}{dE_c} = 2a(\gamma E_B + E_c + \varepsilon)\left(\gamma \frac{d(E - RI)}{dE_c} + 1\right)$$

that is

$$\frac{dI}{dE_c} = \frac{2a(\gamma E_B + E_c + \varepsilon)}{1 + 2a\gamma R(\gamma E + E_c + \varepsilon)}.$$

Multiplying throughout by R, and putting $\gamma = 1/\mu_0$ we obtain by a simple transformation

$$R\frac{dI}{dE_c} = \frac{\mu_0 R}{R + \frac{E_B + \mu_0(E_c + \varepsilon)}{2I}}. \tag{27}$$

Now, $R \cdot dI$ is the voltage change setup in the resistance R, and dE_c is the change in the input voltage. Hence equation (27) gives the voltage amplification produced by the device, which we shall call μ. Furthermore, it follows from paragraph IV, that the output impedance of the amplifier is given by

$$R_0 = \frac{E_B + \mu_0(E_c + \varepsilon)}{2I}. \tag{13}$$

Hence the voltage amplification μ is given by

$$\mu = \frac{\mu_0 R}{R + R_0}. \tag{28}$$

From this equation it is seen that the voltage amplification asymptotically approaches a finite value μ_0, which is attained when the external resistance R becomes infinitely large compared with the output impedance of the amplifier.

In order to find the power amplification it is necessary to know the input impedance of the amplifier; that is, the impedance of the circuit FGE_c (Fig. 4). Now, the amplifier is operated, as was stated above, under such conditions that no current is established in the circuit FGE_c. The impedance of this circuit is, therefore, infinite, and the power developed in it is indeterminate.

In order to give the input circuit a definite constant resistance, Mr. Arnold suggested shunting the filament and grid with a high resistance. This can be considered as the input resistance R_i, of the amplifier. The input voltage is that developed between the ends of this shunt resistance.

If, now, e and e_i represent the voltages established between the ends of the output and input resistances R and R_i respectively, the power developed in R and R_i is e^2/R and e_i^2/R_i. Hence the power amplification is

$$\eta = \frac{e^2}{e_i^2}\frac{R_i}{R} = \mu^2 \frac{R_i}{R}$$

which, with the help of (28), becomes

$$\eta = \frac{\mu_0^2 R_i R}{(R + R_0)^2}. \tag{29}$$

The amplification is, therefore, a maximum when R is equal to R_0.[14]

The power developed in R is

$$P = \frac{\mu_0^2 e_i^2 R}{(R + R_0)^2} \tag{30}$$

from which it follows, as was to be expected, that the power in R is a maximum when the external output resistance R is equal to the output impedance R_0 of the tube.

It is readily seen that the current amplification is given by

$$\xi = \frac{\mu_0 R_i}{R + R_0} \tag{31}$$

from which it follows that the current amplification asymptotically approaches zero as R is increased, the maximum current amplification being obtained when R becomes infinitely small compared with R_0.

Putting $R = R_0$ in (29) and $R = 0$ in (31) and remembering that the slope of the curve giving the relation between plate current and grid voltage is given by

$$S = \frac{\mu_0}{R_0} \tag{15}$$

we get for the maximum power amplification

$$\eta' = \frac{\mu_0 R_i}{4} \cdot S \tag{29a}$$

and for the maximum current amplification

$$\xi' = R_i \cdot S. \tag{31a}$$

These equations show the important part played by the slope S of the curve giving the plate current as a function of the grid voltage. The factor S is equally important in the operation of tube as an oscillation generator and detector. The slope S is what was called by Hazeltine the "mutual conductance" of the tube.[15]

It is seen here that this quantity can be expressed in terms of the two most important and easily determined constants of the tube, namely, the amplification constant μ_0 and the impedance R_0.

While the amplification constant depends only upon the structure of the tube, the impedance, besides being a function of the structure, depends also upon the values of the applied voltages between filament and grid and filament and plate, as is readily seen from the above equation. If the impedance is determined as a function of the plate voltage, the grid voltage being, let us say, zero, a curve is obtained

[14] The amplification equations are derived on the assumption that the external output circuit of the tube contains only pure resistance. When the circuit is reactive, as is common in practice, we can, to a first approximation, substitute the effective impedance for R in the equations. Mathematical proof for this is given by J. R. Carson, PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS, vol. 7, no. 2, April, 1919.

[15] L. A. Hazeltine, PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS, vol. 6, page 63, 1918.

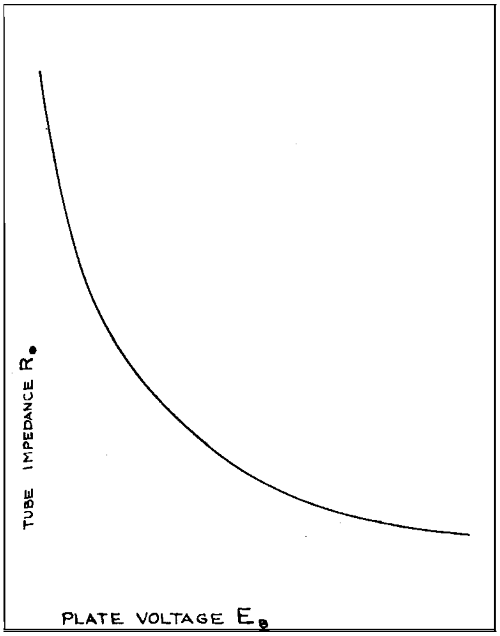

Fig. 10.

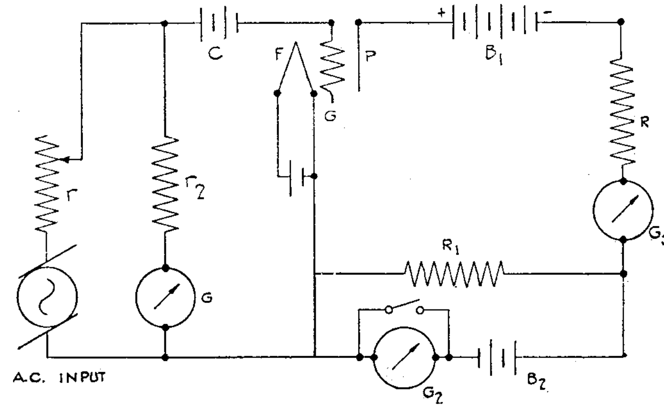

Fig. 11.

somewhat like that shown in Fig. 10. If now it is desired to operate the tube with a definite plate voltage E_B and a grid voltage E_c other than zero, the impedance under these conditions can be obtained from such a curve by adding $\pm\mu_o E_c$ to E_B and reading off the impedance from the curve at a value of the plate voltage equal to $E_B \pm \mu_0 E_c$.

In designing a tube, the structural parameters are so chosen that the tube constants have definite values depending upon the purpose for which the tube is to be used. Suppose it is desired to design a two-stage amplifier set. Since the tube is a potential-operated device, the voltage impressed on the input of the tube must be made as high as possible, irrespective of the value of the current in the input circuit. This is usually done by stepping up the incoming voltage by means of a transformer, the secondary of which is wound to have as high an impedance as possible. For the same reason when the output current of one tube is to be amplified by another, the first tube is made to work into an impedance or resistance which is large compared with its internal output impedance. Such an arrangement allows of a large voltage amplification being obtained from the first tube. This follows from equation (28) which also shows that when used as a voltage amplifier, the tube must have a large amplification constant μ_o. Referring to equation (26), however, it is seen that the larger μ_o is, the smaller is the input voltage e that can be impressed on the tube without producing distortion, provided that E_B is fixed. When it is necessary to use a two- or three-stage amplifier set, the incoming voltage is generally so small that μ_o for the first tube can be quite large and the plate voltage still not excessively high. But then the voltage impressed on the second tube is much larger than that impressed on the first, and the second tube must be so designed as to be capable of handling this voltage. If the first tube, for example, has an amplification constant equal to 40 and works into a resistance four times its own impedance, the voltage on the second tube is 32 times that impressed on the first. It is seen, therefore, that unless the plate voltage on the second tube be made very much higher than that on the first, the two tubes must have entirely different structural parameters, if equation (26) is to be satisfied in both cases. Such considerations show that unless the tubes be properly designed and the plate voltages correctly chosen to satisfy equation (26), there is a practical limit to the number of stages of amplification that can be used. If the limitations imposed by equation (26) are not taken regard of, the process of amplification can result in a considerable amount of distortion, which is a serious matter when using the device for amplifying telephonic currents. When using it for telegraph purposes, such as the amplification of radio telegraph signals at the receiving station, the distortion produced results in a waste of energy in harmonics.

VIII. Experimental Verification of Amplification Equations

The circuit shown in Fig. 11 is not of the type customarily used in practice, but was designed to test the equations developed in the previous paragraph. This type of circuit was necessitated by the following reasons. Referring to equation (24), it is seen that if the input voltage $e \sin pt$ is zero, the current thru the tube is given by the first term of the equation, which is larger than the alternating current term. For finite values of $e \sin pt$, the resulting alternating current established in the output circuit, which is to be measured, cannot be separated in the usual way from this direct current with the help of appropriate inductances and capacities, since then the measured amplification would be largely determined by the constants of the circuit. On the other hand, it is not possible to make the amplifier work simply into a straight noninductive resistance alone, since the direct current that would flow thru the galvanometer is in most cases large compared with the output alternating current, so that a galvanometer which would be capable of carrying the direct current would not be sensitive enough to measure the output alternating current with any degree of accuracy. This was overcome by using a balancing circuit shown in Fig. 11. R and R_1' are two noninductive

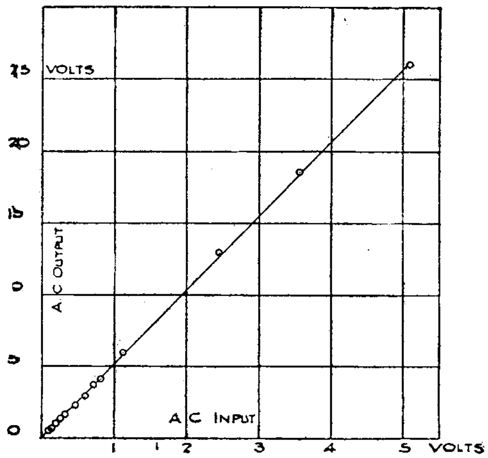

Fig. 12.

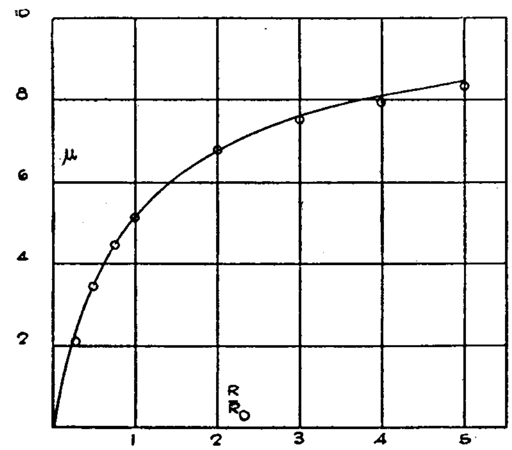

Fig. 13. $\mu_o = 10.2$, input $= 0.45$ volt.

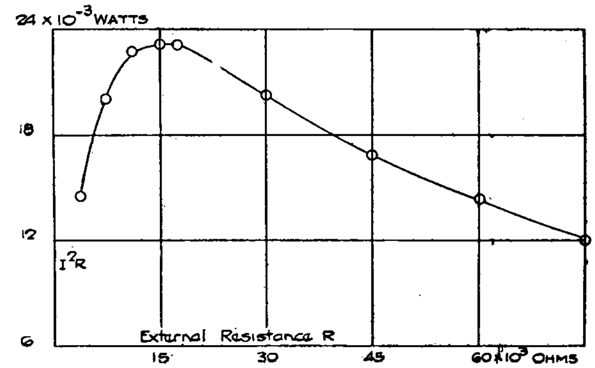

Fig. 14.

resistances stretched upon a board. Parallel to R_1 was shunted a sensitive alternating current galvanometer, G_2, and a balancing battery B_2. This battery was so adjusted that when no alternating current input was applied to the tube, the current thru the galvanometer was zero, that is, the direct current in the output circuit went thru the resistance R_1. This resistance was large compared with that of the galvanometer; hence practically all the alternating current established in the output when the input voltage was impressed, went thru the galvanometer G_2. The effective resistance into which the tube worked was, of course, given by R. The input was varied with the help of the resistances r_1 and r_2. The whole system was carefully shielded and care was taken to avoid any effects due to shunt and mutual capacity of the leads and resistances. The input voltage was varied from a few hundredths of a volt to several volts and the frequency from 200 to 350 000 cycles per second.

Some of the results are shown in the following figures. Fig. 12 shows the output voltage (that is, the voltage across the external resistance R) as a function of the input voltage for a frequency of 1000 cycles per second. The linear relation indicates that the voltage amplification is independent of the input voltage; hence also the power amplification is independent of the input power.

Fig. 13 shows the results obtained when the voltage amplification was measured as a function of the external resistance R. The circles show the observed values, while the curves were calculated from equation (28). It is seen that the agreement is quite good. In this case the input voltage was 0.45 volt and the value of μ_o for this tube, as measured by the direct current method explained in paragraph V. was 10.2.

In another experiment, the output power was determined as a function of the external resistance. According to equation (30) this should be a maximum when the external resistance R is equal to the impedance R_o of the tube. In this case the input voltage e_1 was 3.55 volts, and $\mu_o = 10.2$. The impedance of the tube was kept constant at 14 800 ohms. This was done by always adjusting the plate voltage so that when the external resistance was changed the current thru the tube was kept constant. From the results given in Fig. 14 it is seen that the maximum occurs at $R = 15\,000$, which is very nearly equal to the impedance of the tube. This result is in accordance with equation (30). Furthermore, the maximum power computed from equation (30) is 22.2 10^{-3} watt, which is sufficiently close to the observed value, 23 10^{-3} watt, to verify equation (30). This equation does not give the maximum power that can be handled by the tube but merely the power developed in the external resistance R for a given value of the input voltage e_i. The maximum power is obtained when the input voltage has the value given by equation (26).

The circuit shown in Fig. 11 was used merely to test the equation derived in this paper. It is not suitable for practical purposes where it is necessary to test a large number of tubes with the speed and facility called for in practice. If the tube is to be used as a voltage amplifier all that is necessary is a determination of the amplification constant μ_o. The actual voltage amplification obtainable in any particular circuit can then be deduced from equation (28). When the tube is to be used as a power amplifier, a transmission test is used which is common in telephone practice. If it is desired to operate the tube as power amplifier in a certain circuit, its amplification is tested in an equivalent circuit which is arranged so that a note of, say, 800 cycles can be transmitted either straight to a telephone receiver or, by throwing a switch, to the receiver thru the tube and an artificial telephone line, the attenuation of which can be adjusted until the note heard in the receiver is of the same intensity for both positions of the switch. When this is the case the amplification given by the tube is equal to the attenuation produced by the line. The attenuation can be computed from the constants of the line and is usually expressed in terms of miles length of cable of specified constants. This method of measuring and expressing the amplification is convenient in practice. But it must be remembered that this notation has very little meaning and is apt to lead to confusion unless the constants of the cable are definitely specified or previously agreed upon. It simply means that the amplification is equivalent to the attenuation which would be produced by so many miles of a certain sort of cable having a certain definite attenuation constant. Thus, the current amplification produced by the tube can be expressed in terms of length, d, of cable by the following equation:

$$d = K \log_{10} \frac{i_2}{i_1}.$$

where K is determined by the attenuation of that cable. When dealing with power amplification K must be divided by two. If we adopt as standard the so-called "standard number 19 gauge cable" used by the Western Electric Company, the length, d, is expressed in miles when the constant, K, has the value $K = 21.13$. The fact that this constant is already finding its way into common vacuum practice would suggest its general adoption when speaking of the amplification of a tube. On the other hand, since the unit of measurement is not a cable but a constant, it might have been more desirable to adopt for K the simple value 20, which would have simplified computations. However, the main point is that it is very important to have a common agreement on the value of K.

The foregoing considerations show that the structural parameters play a very important part in the operation of the tube. On them depend the constants μ_o and R_o which appear in the amplification equations and which are involved explicitly and implicitly in the fundamental equation of the characteristic (equation 6). Proper structural design manifests many latent possibilities of this type of device, and enables us to meet the many conditions that must be complied with in order to obtain satisfactory operation in its ever-increasing number of applications. By proper choice of the structural parameters, tubes have been designed to have voltage and power amplification covering a wide range. A power amplification of 3000-fold was found possible with a single tube using a plate voltage of only 100 volts. It is not difficult to obtain a voltage amplification of several hundred fold, but in building tubes of such high voltage amplification regard must be taken of the increase in impedance with increase in μ_o, as shown by equations (12) and (13).

Although the simple theory of operation given in this paper applies specifically to the case in which the tube is used as an amplifier, it has also been of considerable help in designing and developing vacuum tube oscillation generators and detectors. The design of a good detector tube depends very much upon operating conditions. The detecting qualities of a tube can easily be increased by designing it to operate on comparatively high voltages. This is sometimes desirable in radio stations where high voltages are available and it is desired to use the heterodyne method of reception. On the other hand, it is often important to use tubes of such design that they can operate efficiently on low voltages and with small power consumption. In this connection it may be said that detectors have been designed to give satisfactory operation with two volts on the filament and a plate voltage of 6 volts and less. In the absence of any satisfactory way of expressing the efficiency of a detector, it is unfortunately not possible to say just how good such a detector is. The problem of measuring the detecting efficiency will be reserved for a future paper.

Acknowledgment

The indebtedness of the writer is due to Mr. E. H. Colpitts and Mr. H. D. Arnold for valuable advice and kind interest which greatly facilitated the work; and to Mr. H. W. Everitt for able assistance in carrying out the experiments.